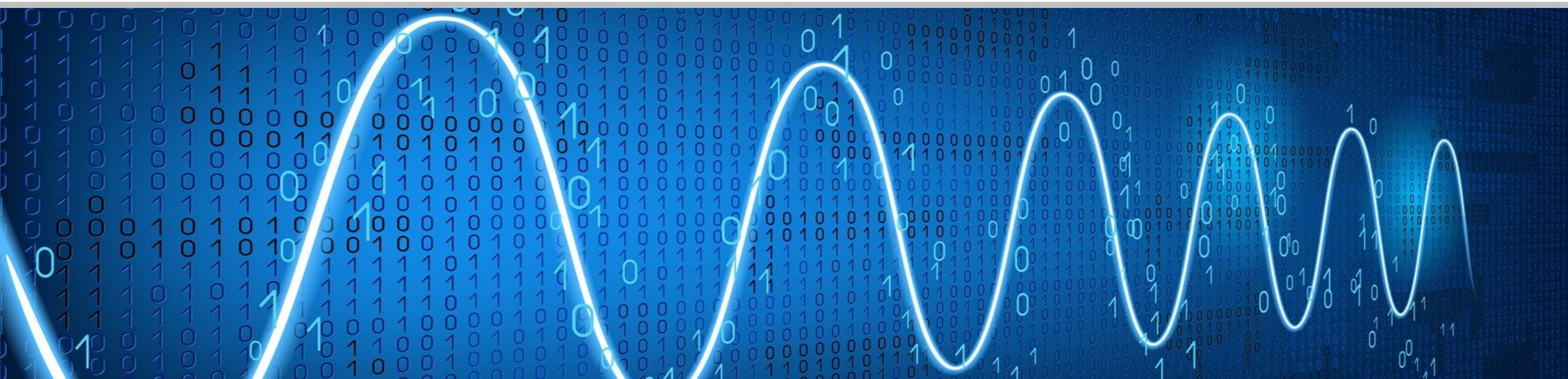
Digital Signal Processing

Lab 04: Signal Representation and Modeling II

Abdallah El Ghamry



Signal Representation and Modeling

The purpose of this lab is to

- Understand the concept of a signal and how to work with mathematical models of signals.
- Discuss fundamental signal types and signal operations used in the study of signals and systems.
- Experiment with methods of simulating continuous- and discrete-time signals with MATLAB.
- Discuss symmetry properties.
- Explore characteristics of sinusoidal signals.
- Learn energy and power definitions.

Basic Building Blocks For Continuous-Time Signals

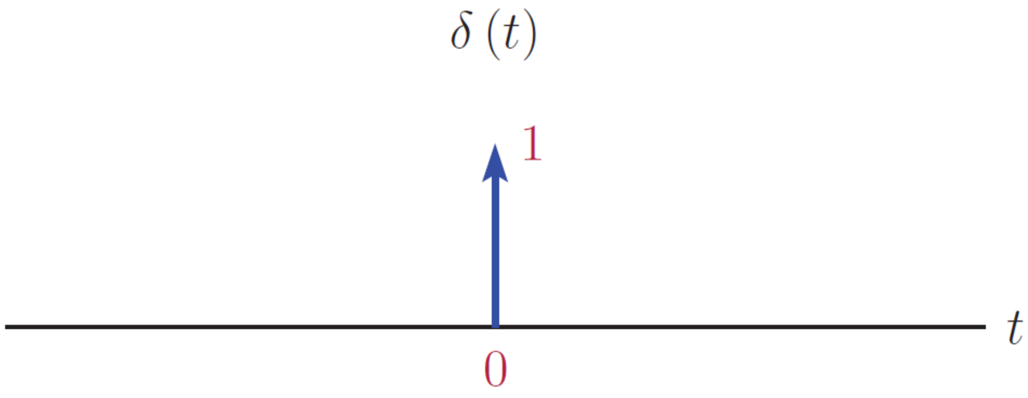
- There are certain basic signal forms that can be used as building blocks for describing signals with higher complexity.
- In this section we will study some of these signals.
- Mathematical models for more advanced signals can be developed by combining these basic building blocks through the use of the signal operations described before.

Unit-Impulse Function

- The unit-impulse function plays an important role in mathematical modeling and analysis of signals and linear systems.

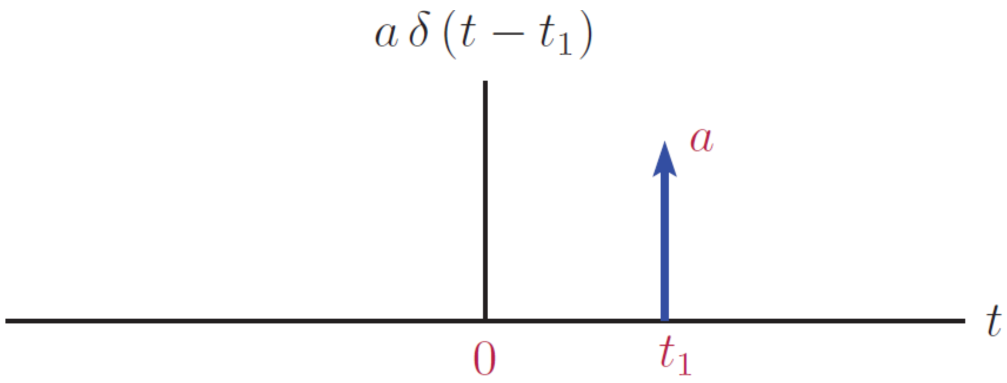
$$\delta(t) = \begin{cases} 0\,, & \text{if } t \neq 0 \\ \text{undefined}\,, & \text{if } t = 0 \end{cases}$$

$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$



$$a\,\delta(t - t_1) = \begin{cases} 0\,, & \text{if } t \neq t_1 \\ \text{undefined}\,, & \text{if } t = t_1 \end{cases}$$

$$\int_{-\infty}^{\infty} a\,\delta(t - t_1)\, dt = a$$



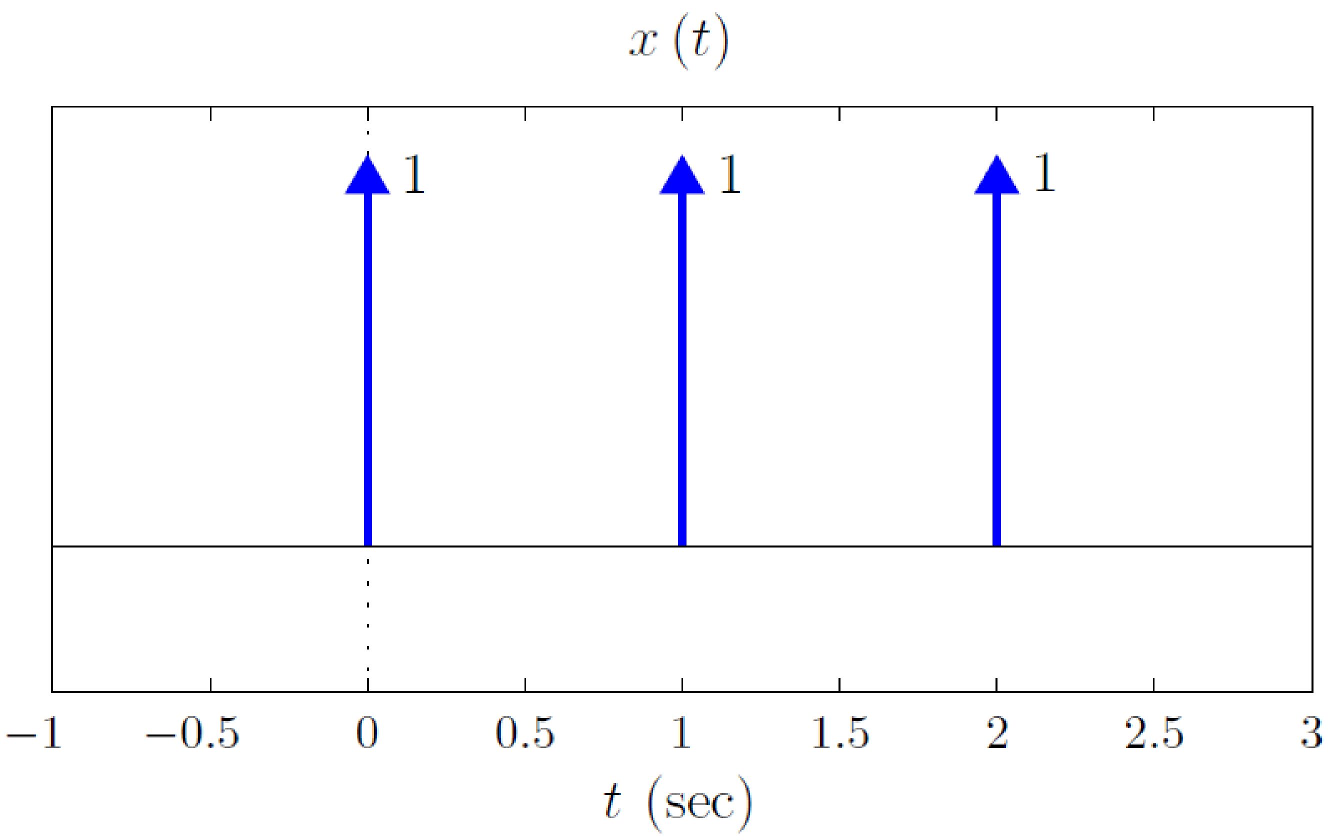
Problem 1.8 (a)

1.8. Sketch each of the following functions.

a. $\delta(t) + \delta(t-1) + \delta(t-2)$

Problem 1.8 (a) – Solution

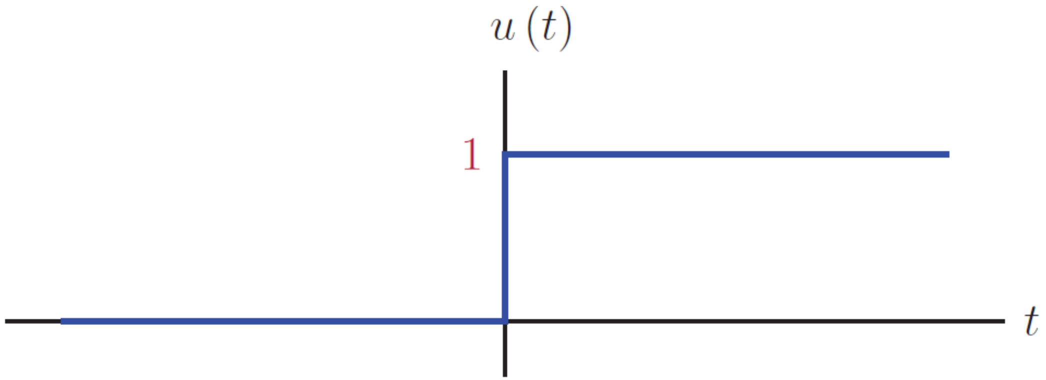
$$\delta(t) + \delta(t-1) + \delta(t-2)$$



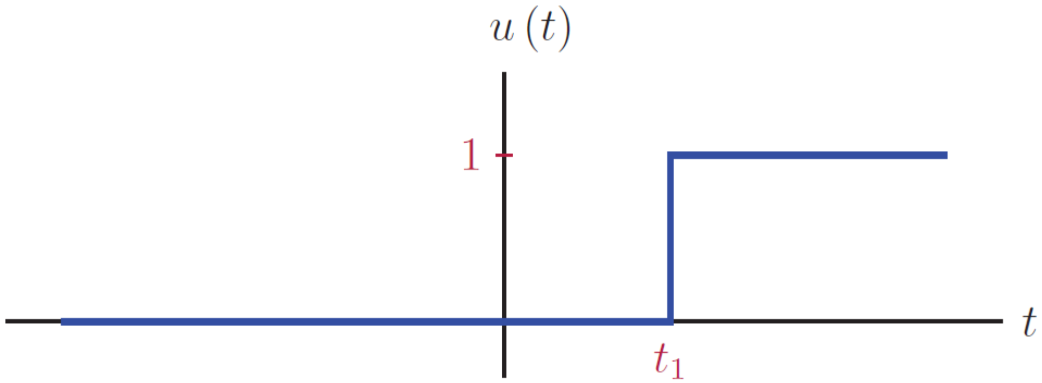
Unit-Step Function

- The unit-step function is useful in situations where we need to model a signal that is turned on or off at a specific time instant.

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



$$u(t - t_1) = \begin{cases} 1, & t > t_1 \\ 0, & t < t_1 \end{cases}$$



Problem 1.9 (a)

1.9. Sketch each of the following functions in the time interval $-1 \leq t \leq 5$.

a. $u(t) + u(t-1) - 3u(t-2) + u(t-3)$

Problem 1.9 (a) – Solution

a. $u(t) + u(t-1) - 3u(t-2) + u(t-3)$

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

$$u(t-1) = \begin{cases} 1, & t > 1 \\ 0, & t < 1 \end{cases}$$

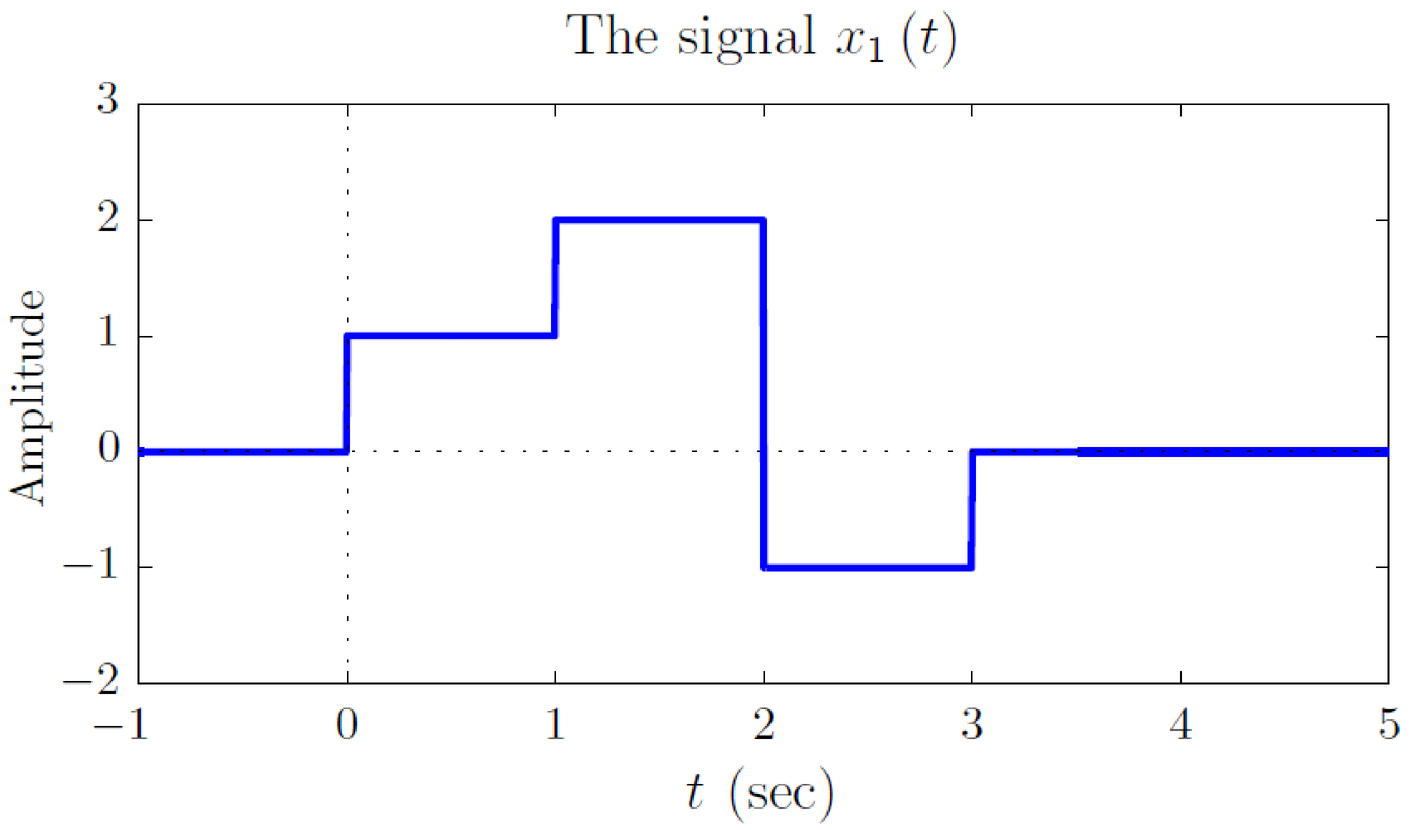
$$-3u(t-2) = \begin{cases} -3, & t > 2 \\ 0, & t < 2 \end{cases}$$

$$u(t-3) = \begin{cases} 1, & t > 3 \\ 0, & t < 3 \end{cases}$$

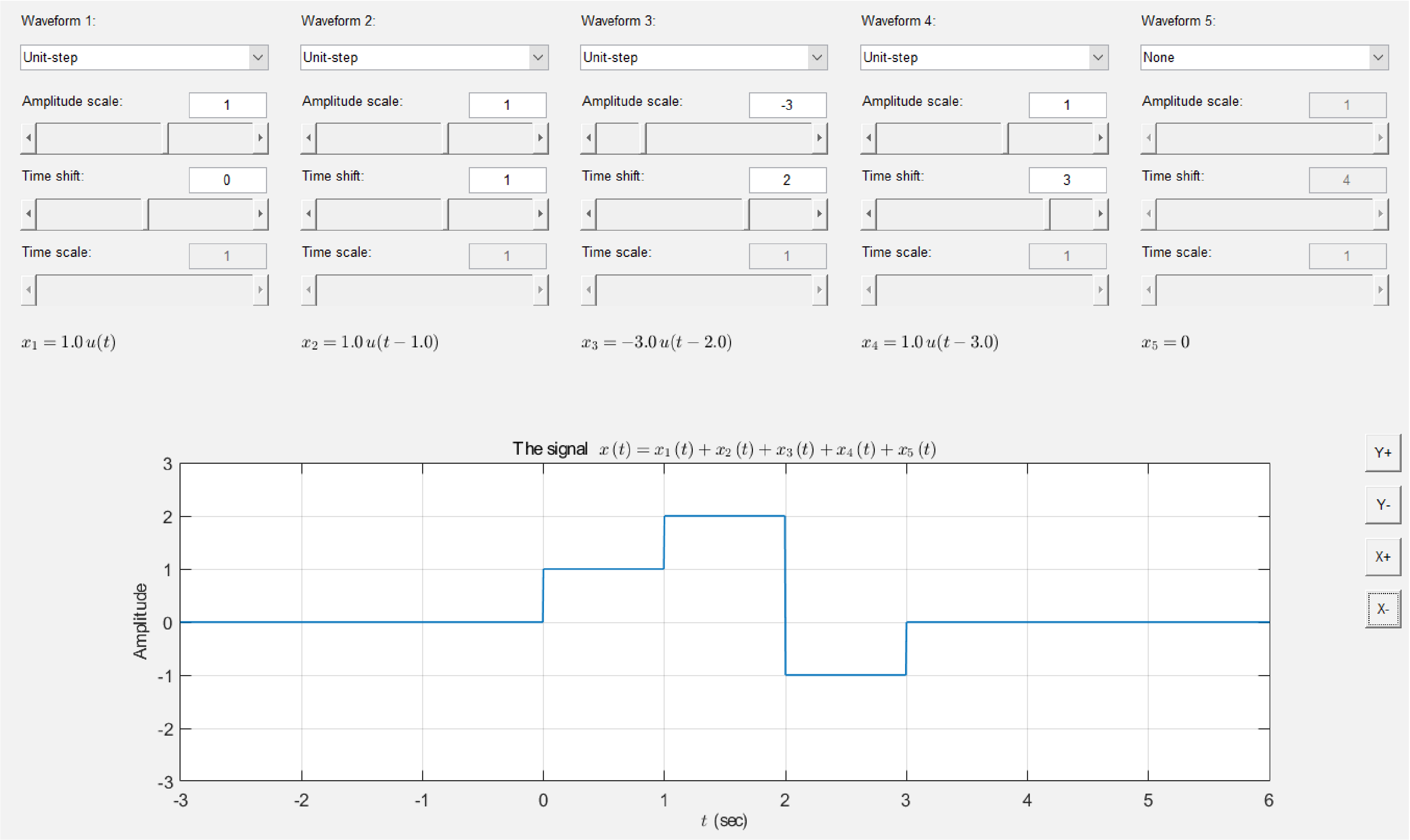
$$u = \begin{cases} 0, & t < 0 \\ 1, & 0 < t < 1 \\ 2, & 1 < t < 2 \\ -1, & 2 < t < 3 \\ 0, & t > 3 \end{cases}$$

Problem 1.9 (a) – Solution

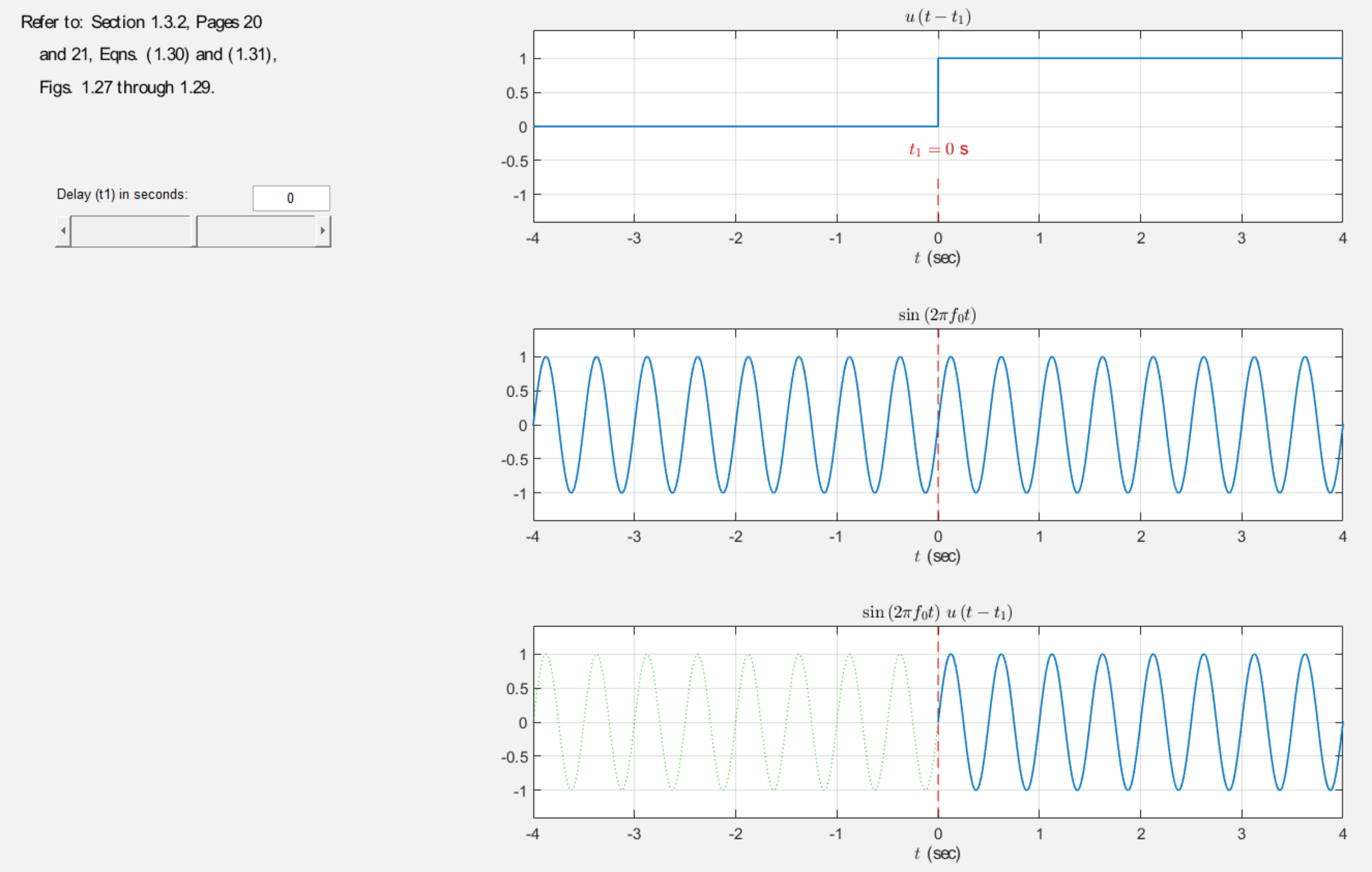
a. $$u(t) + u(t-1) - 3u(t-2) + u(t-3)$$



Problem 1.9 (a): wav_demo1



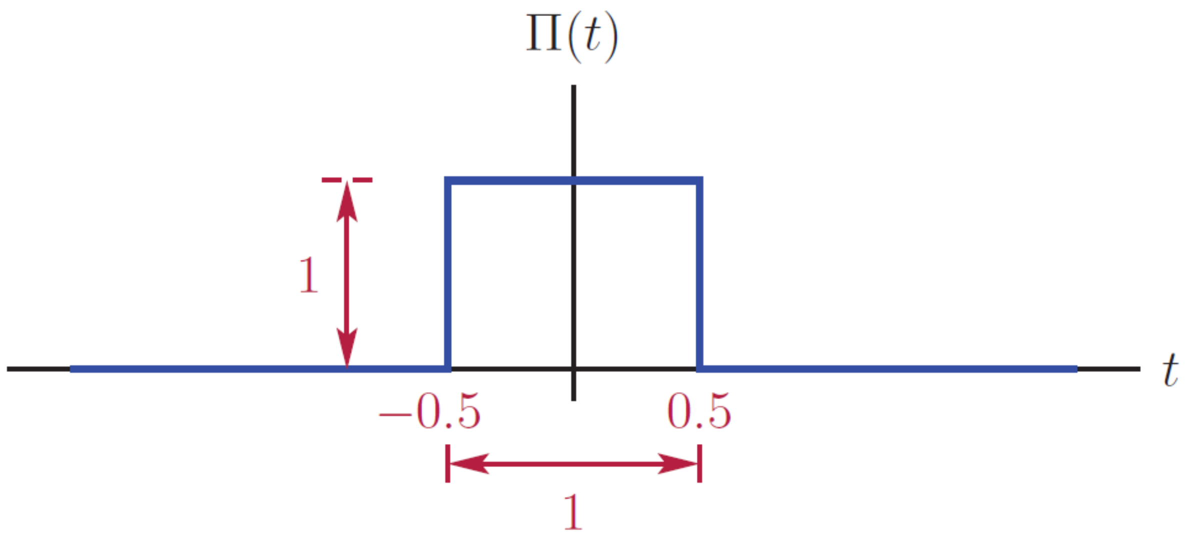
Unit-Step Function: stp_demo1



Unit-Pulse Function

- We will define the unit-pulse function as a rectangular pulse with unit width and unit amplitude, centered around the origin.

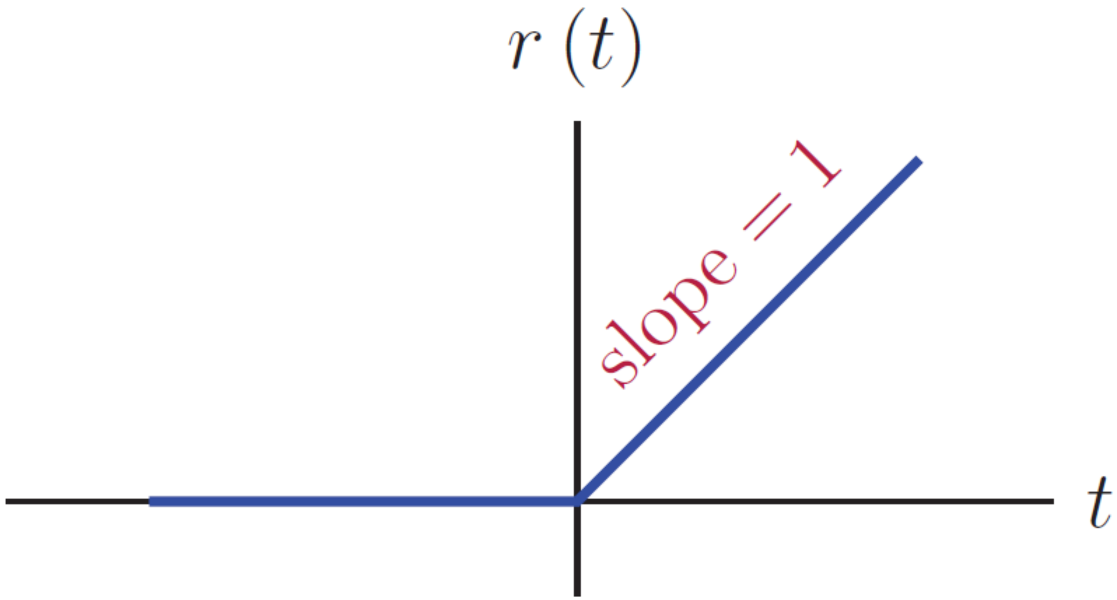
$$\Pi(t) = \begin{cases} 1, & |t| < \frac{1}{2} \\ 0, & |t| > \frac{1}{2} \end{cases}$$



Unit-Ramp Function

- The unit-ramp function has zero amplitude for t < 0, and unit slope for t ≥ 0.

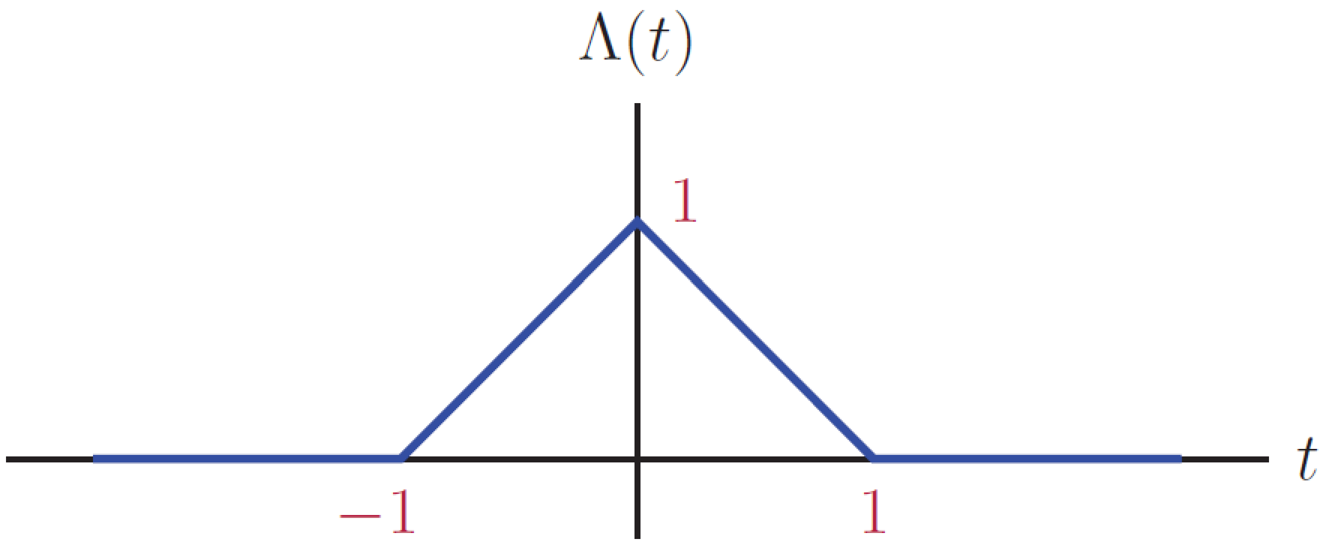
$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Unit-Triangle Function

- The unit-triangle function is defined as

$$\Lambda(t) = \begin{cases} t+1, & -1 \le t < 0 \\ -t+1, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$



Problem 1.9 (e)

1.9. Sketch each of the following functions in the time interval $-1 \leq t \leq 5$.

e. $\Lambda(t) + 2\Lambda(t-1) + 1.5\Lambda(t-3) - \Lambda(t-4)$

Problem 1.9 (e) – Solution

e. $\Lambda(t) + 2\Lambda(t-1) + 1.5\Lambda(t-3) - \Lambda(t-4)$

$$\Lambda(t) = \begin{cases} t+1, & -1 \le t < 0 \\ -t+1, & 0 \le t < 1 \\ 0, & \text{otherwise} \end{cases}$$

$$2\Lambda(t-1) = \begin{cases} 2t, & 0 \le t < 1 \\ -2t+4, & 1 \le t < 2 \\ 0, & \text{otherwise} \end{cases}$$

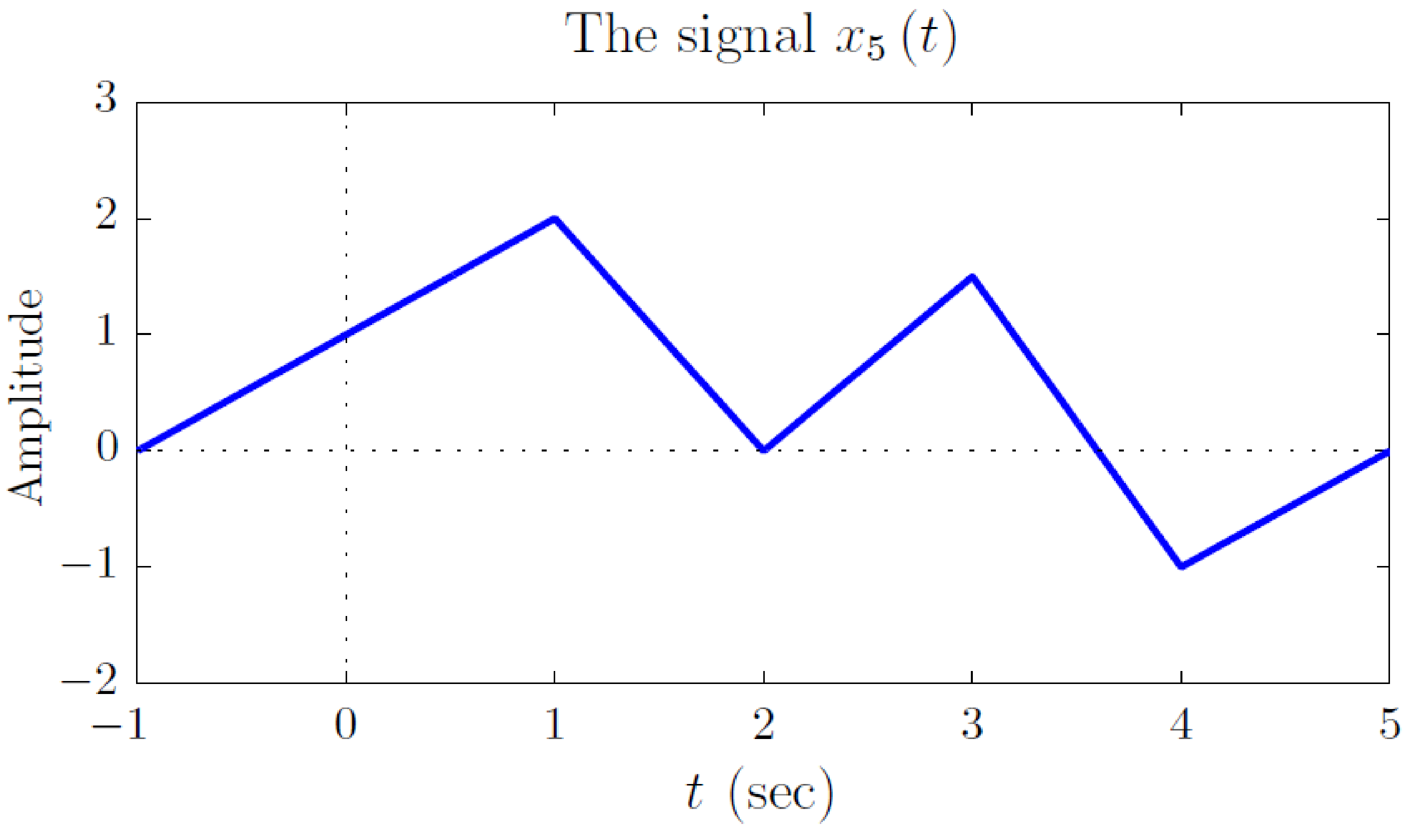
$$1.5\Lambda(t-3) = \begin{cases} 1.5t-3, & 2 \le t < 3 \\ -1.5t+6, & 3 \le t < 4 \\ 0, & \text{otherwise} \end{cases}$$

$$-\Lambda(t-4) = \begin{cases} -t+3, & 3 \le t < 4 \\ t-5, & 4 \le t < 5 \\ 0, & \text{otherwise} \end{cases}$$

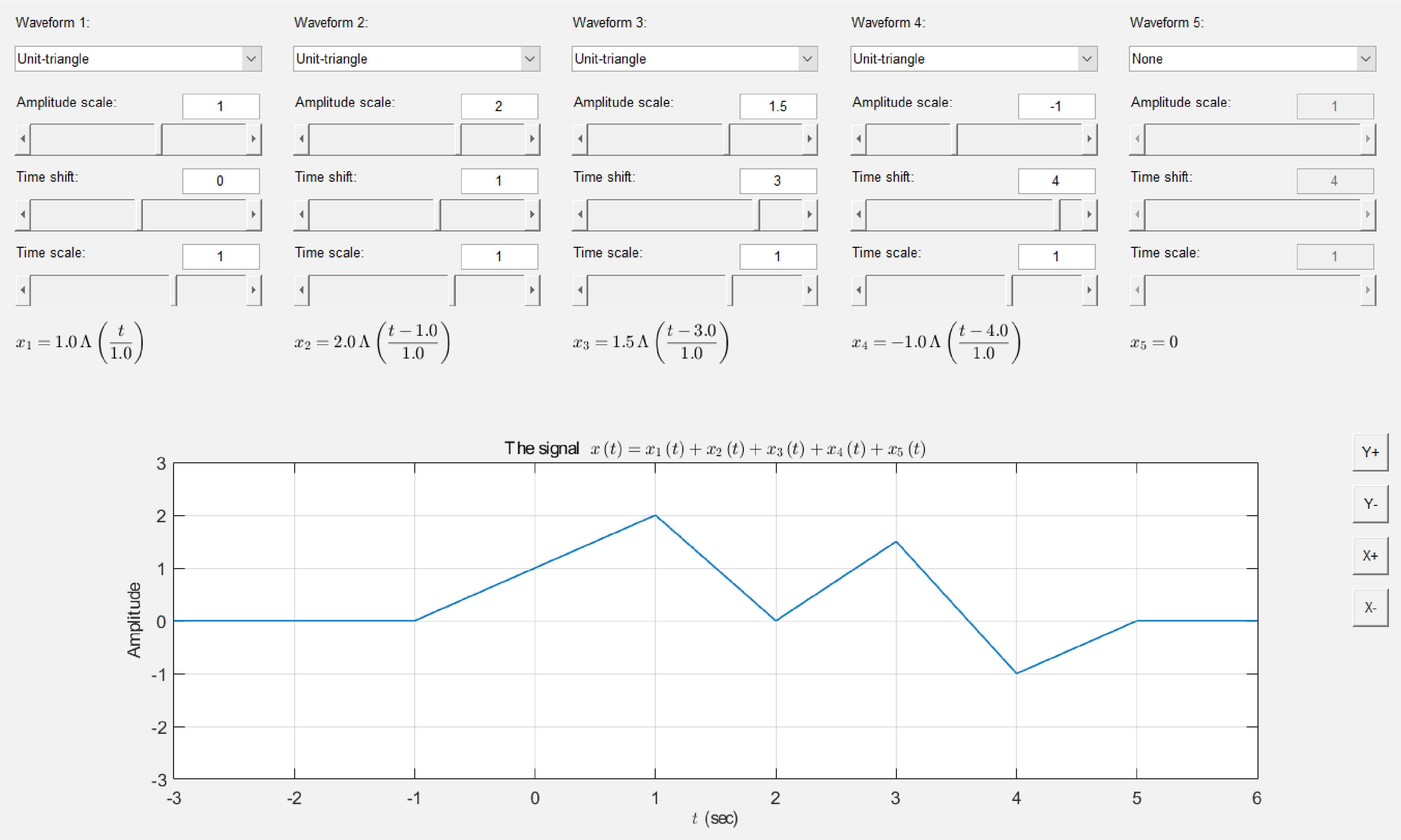
$$\Lambda = \begin{cases} t+1, & -1 \le t < 0 \\ t+1, & 0 \le t < 1 \\ -2t+4, & 1 \le t < 2 \\ 1.5t-3, & 2 \le t < 3 \\ -2.5t+9, & 3 \le t < 4 \\ t-5, & 4 \le t < 5 \\ 0, & \text{otherwise} \end{cases}$$

Problem 1.9 (e) – Solution

e. $$\Lambda(t) + 2\Lambda(t-1) + 1.5\Lambda(t-3) - \Lambda(t-4)$$



Problem 1.9 (e): wav_demo1



Sinusoidal Signals

- The general form of a sinusoidal signal is

$$x(t) = A\cos(\omega_0 t + \theta)$$

- The parameter A is the amplitude of the signal.
- The parameter ω_0 is the radian frequency which has the unit of rad/s.

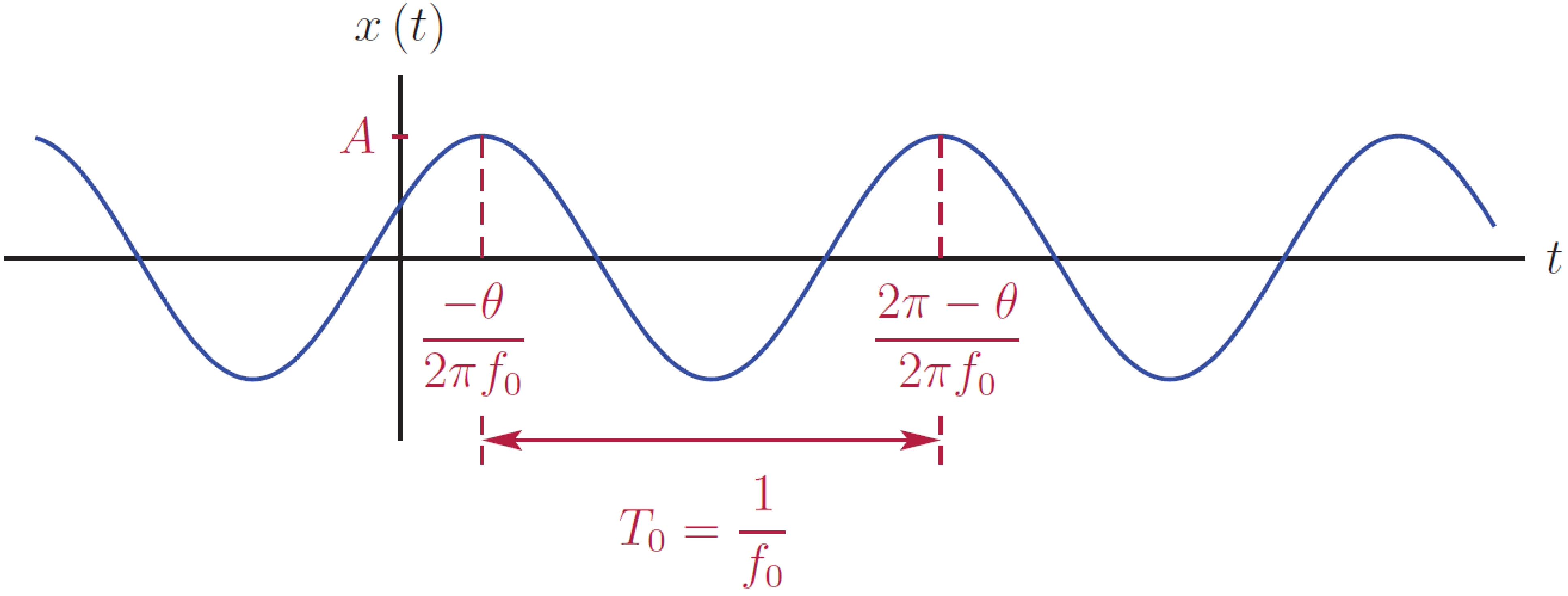
$$\omega_0 = 2\pi f_0$$

- The parameter θ is the initial phase angle in radians.

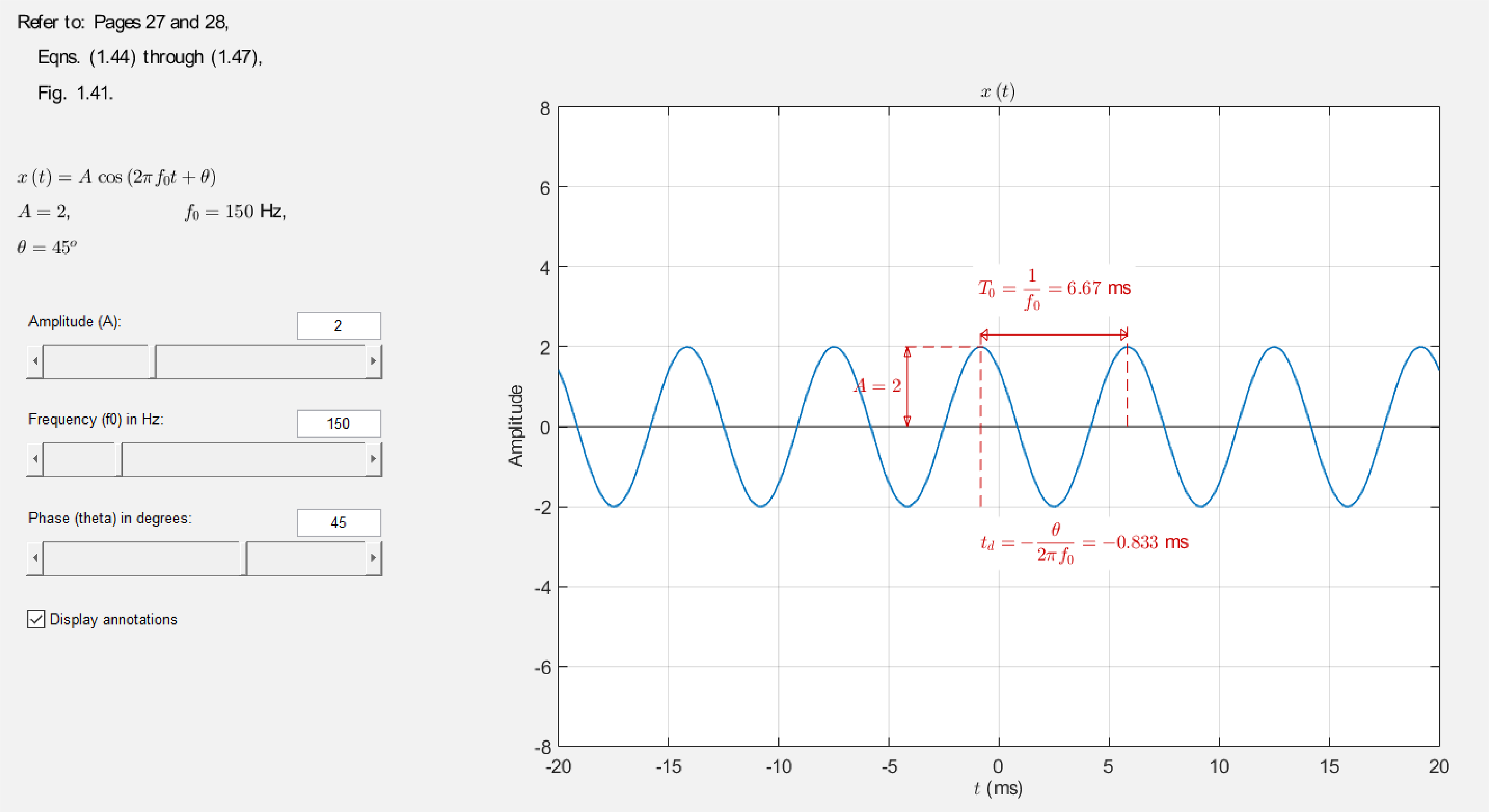
Sinusoidal Signals

- The amplitude parameter A controls the peak value of the signal.

$$x(t) = A\cos(\omega_0 t + \theta)$$



Sinusoidal Signals: sin_demo1



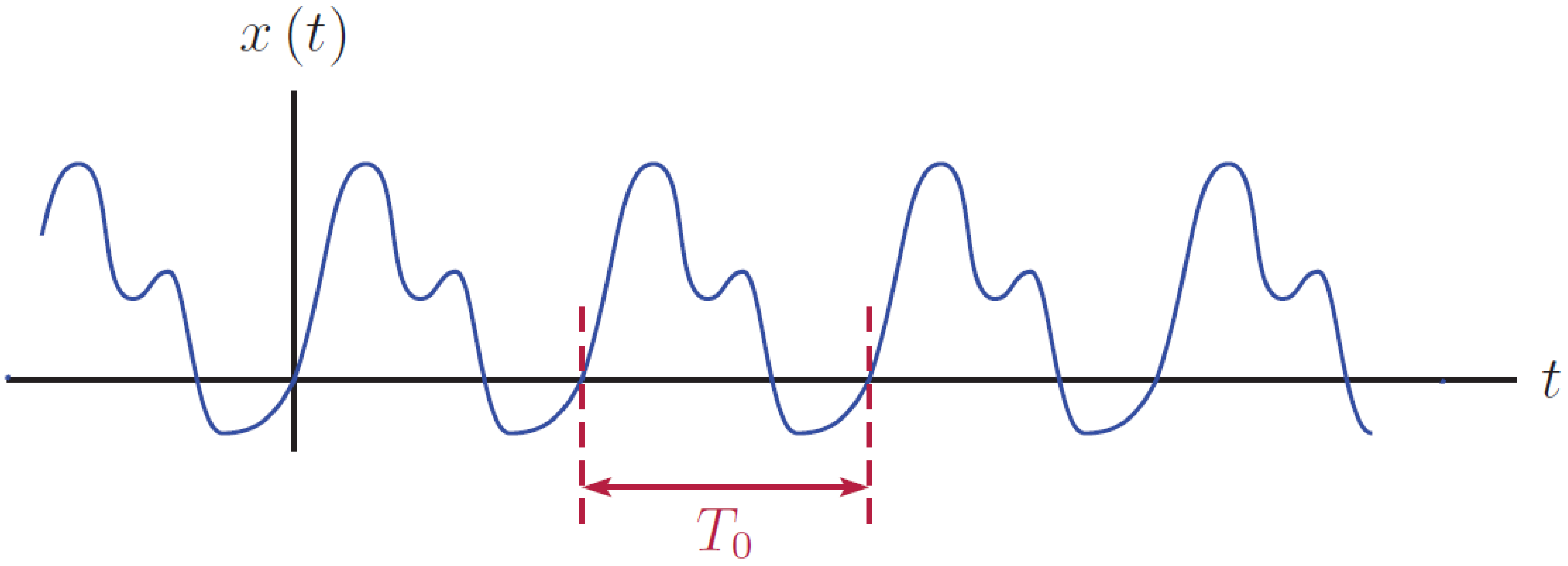
Periodic vs. Non-Periodic Signals

- A signal is said to be periodic if it satisfies

$$x(t + T_0) = x(t)$$

at all time instants t, and for a specific value of $T_0 \neq 0$.

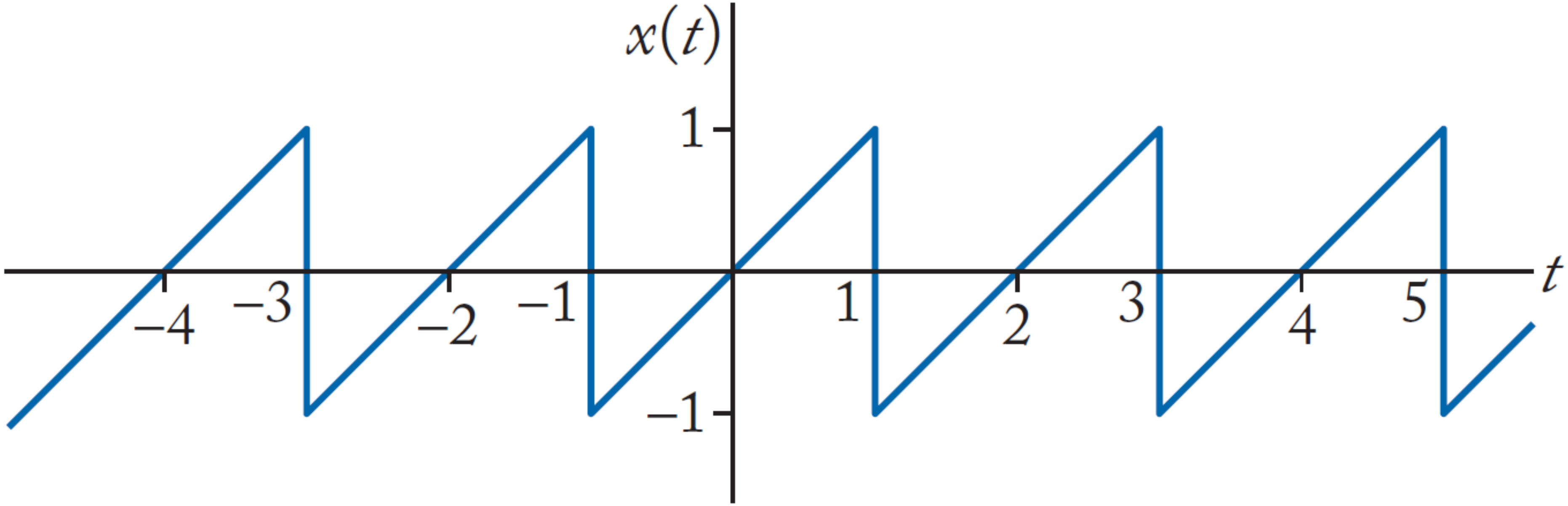
- The value T_0 is referred to as the period of the signal.



Periodic Signals

- A signal is said to be periodic if it satisfies

$$x(t + T_0) = x(t)$$



- The period T_0 is 2.

Euler's Formula

- A complex exponential function can be expressed in the form

$$e^{jx} = \cos(x) + j\,\sin(x)$$

- This relationship is known as Euler's formula.
- It will be used extensively in working with signals, linear systems and various transforms.

Problem 1.17

1.17. Using the definition of periodicity, determine if each signal below is periodic or not. If the signal is periodic, determine the fundamental period and the fundamental frequency.

b. $x\left(t\right) = 2\,\sin\left(\sqrt{20}t\right)$

g. $x\left(t\right) = e^{j(2t+\pi/10)}$

Problem 1.17 (b) – Solution

b. $x(t) = 2\sin\left(\sqrt{20}t\right)$

b. Periodic.

$$2\pi f_0 = \sqrt{20} \quad \Rightarrow \quad f_0 = \frac{\sqrt{20}}{2\pi} = \frac{\sqrt{5}}{\pi}\ \text{Hz}\ , \qquad T_0 = \frac{1}{f_0} = \frac{\pi}{\sqrt{5}}\ \text{sec}$$

Problem 1.17 (g) – Solution

g. $$x(t) = e^{j(2t+\pi/10)}$$

g. Periodic.

$$x(t) = cos(2t + \pi/10) + j\,sin(2t + \pi/10)$$

$$2\pi f_0 = 2 \quad \Rightarrow \quad f_0 = \frac{1}{\pi}\ \text{Hz}\ , \quad T_0 = \frac{1}{f_0} = \pi\ \text{sec}$$

Example 1.6

Discuss the periodicity of the signals

a. $$x(t) = \sin(2\pi\, 1.5\, t) + \sin(2\pi\, 2.5\, t)$$

b. $$y(t) = \sin(2\pi\, 1.5\, t) + \sin(2\pi\, 2.75\, t)$$

Example 1.6 (a) – Solution

$$x(t) = \sin(2\pi\, 1.5\, t) + \sin(2\pi\, 2.5\, t)$$

a. For this signal, the fundamental frequency is $f_0 = 0.5$ Hz. The two signal frequencies can be expressed as

$$f_1 = 1.5 \text{ Hz } = 3 f_0 \quad \text{and} \quad f_2 = 2.5 \text{ Hz } = 5 f_0$$

The resulting fundamental period is $T_0 = 1/f_0 = 2$ seconds. Within one period of $x(t)$ there are $m_1 = 3$ full cycles of the first sinusoid and $m_2 = 5$ cycles of the second sinusoid. This is illustrated in Fig. 1.45.

Example 1.6 (a) – Periodicity

$$x\,(t) = \sin\,(2\pi\,1.5\,t) + \sin\,(2\pi\,2.5\,t)$$

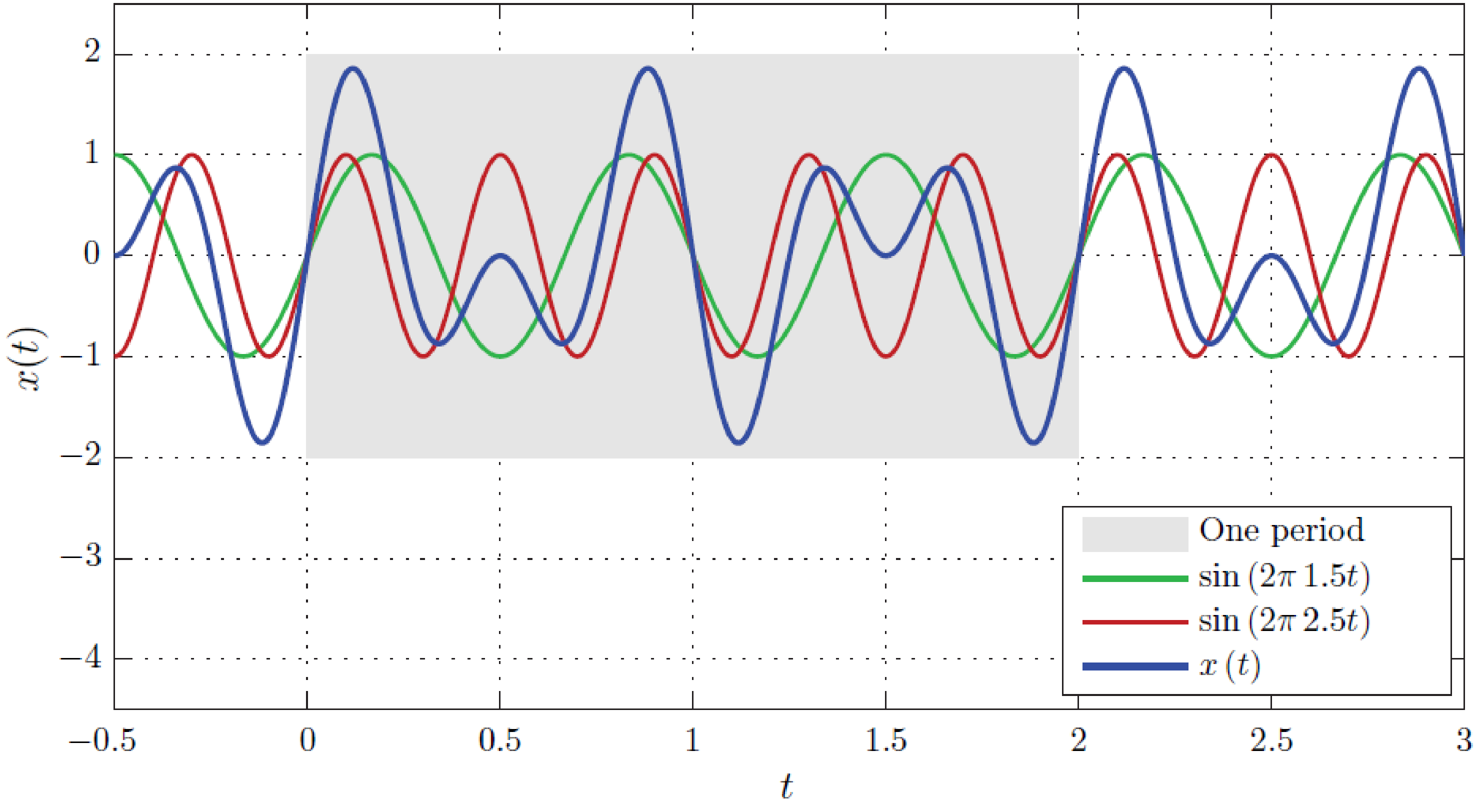


Figure 1.45 – Periodicity of $x\,(t)$ of Example 1.6.

Example 1.6 (b) – Solution

$$y(t) = \sin(2\pi\, 1.5\, t) + \sin(2\pi\, 2.75\, t)$$

b. In this case the fundamental frequency is $f_0 = 0.25$ Hz. The two signal frequencies can be expressed as

$$f_1 = 1.5 \text{ Hz } = 6 f_0 \quad \text{and} \quad f_2 = 2.75 \text{ Hz } = 11 f_0$$

The resulting fundamental period is $T_0 = 1/f_0 = 4$ seconds. Within one period of $x(t)$ there are $m_1 = 6$ full cycles of the first sinusoid and $m_2 = 11$ cycles of the second sinusoid. This is illustrated in Fig. 1.46.

Example 1.6 (b) – Periodicity

$$y(t) = \sin(2\pi\, 1.5\, t) + \sin(2\pi\, 2.75\, t)$$

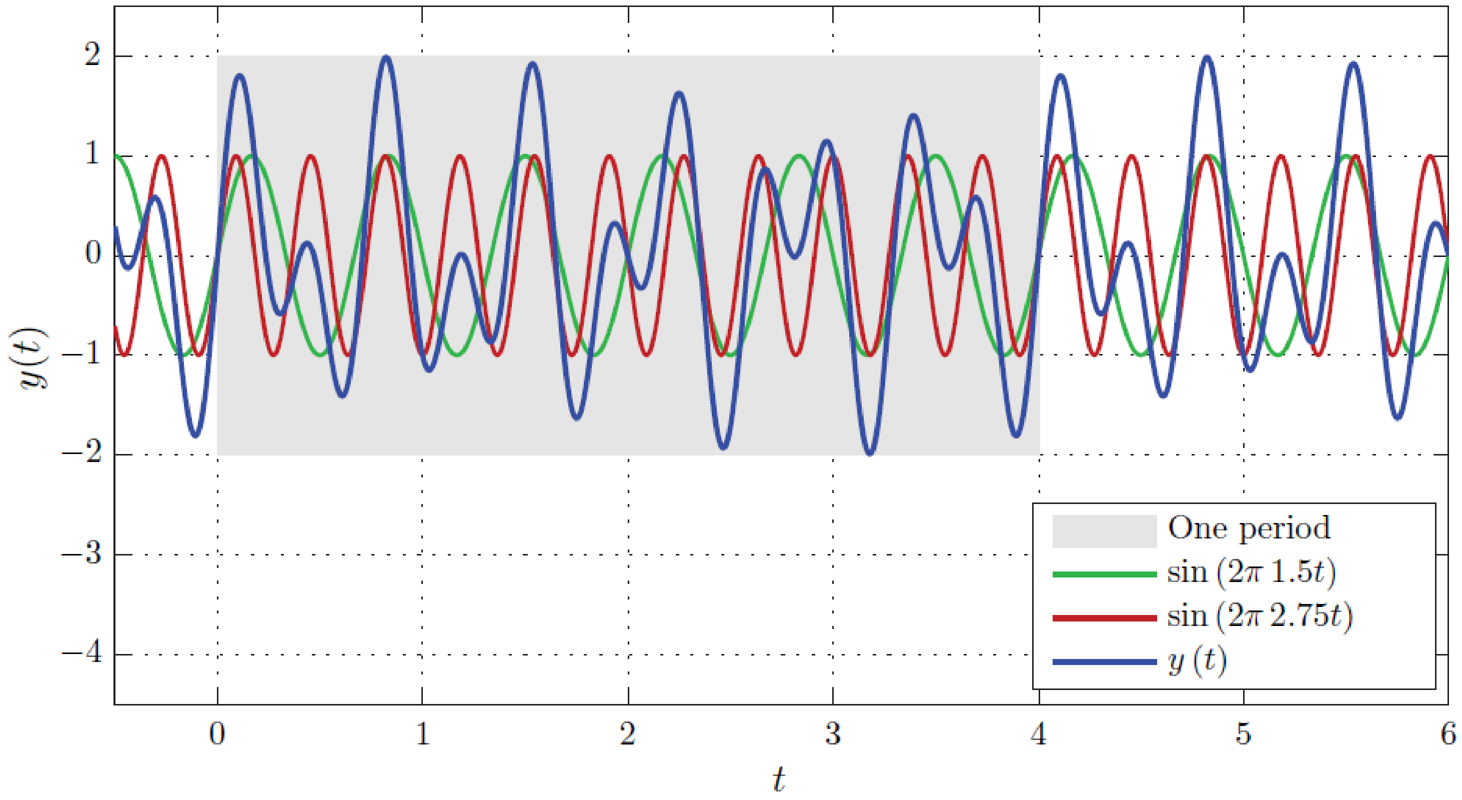


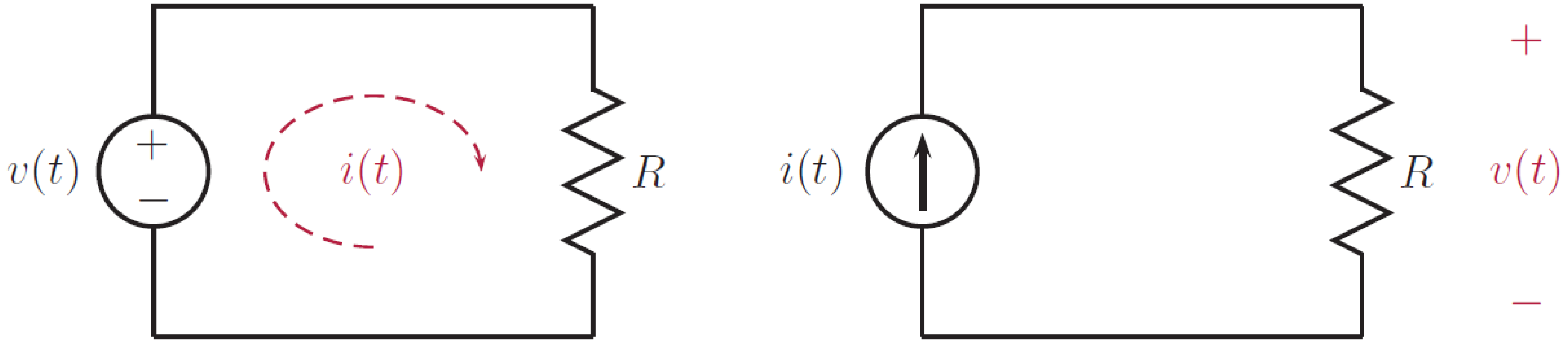
Figure 1.46 – Periodicity of $y(t)$ of Example 1.6.

Energy of a Signal

- We will define the normalized energy of a real-valued signal $x(t)$ as

$$E_x = \int_{-\infty}^{\infty} x^2(t)\, dt$$

- Consider a voltage source with voltage $v(t)$ connected to the terminals of a resistor with resistance R.
- Let $i(t)$ be the current that flows through the resistor.



Energy of a Signal

- The total energy dissipated in the resistor would be

$$E = \int_{-\infty}^{\infty} v(t)\, i(t)\, dt = \int_{-\infty}^{\infty} \frac{v^2(t)}{R}\, dt$$

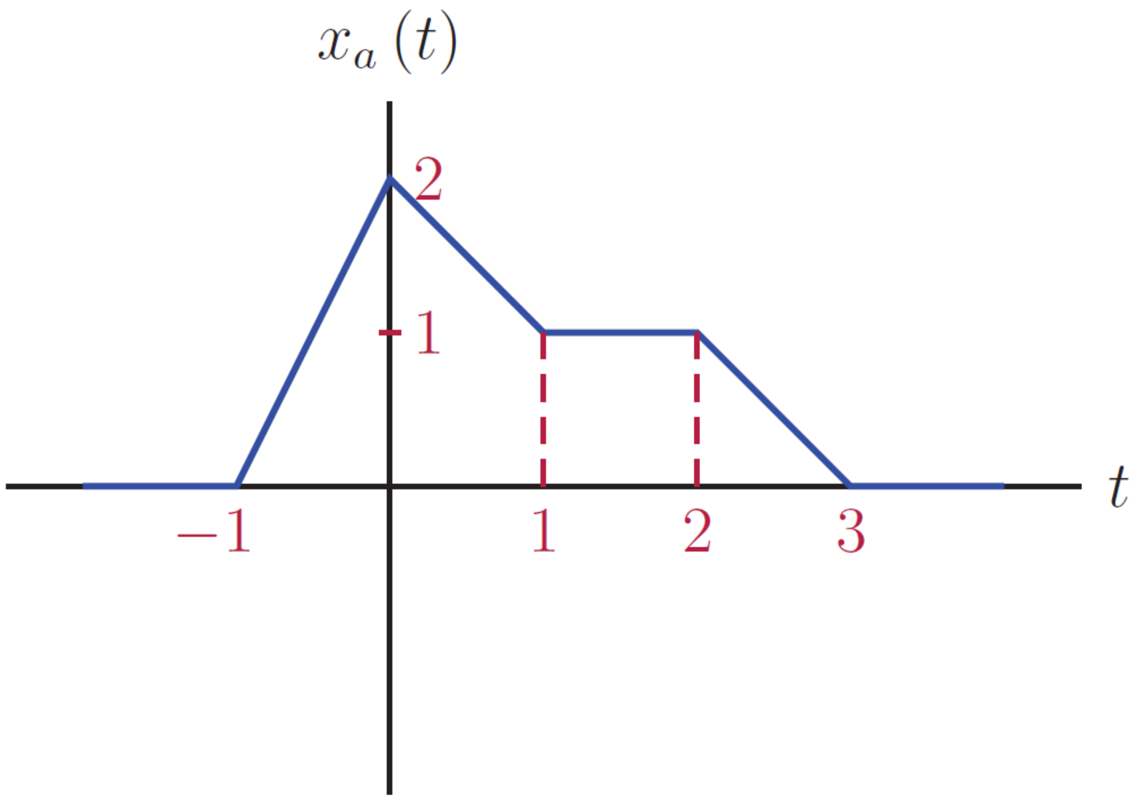
$$E = \int_{-\infty}^{\infty} v(t)\, i(t)\, dt = \int_{-\infty}^{\infty} R\, i^2(t)\, dt$$

- If the resistor value is chosen to be $R = 1\Omega$, then both equations would produce the same numerical value:

$$E = \int_{-\infty}^{\infty} \frac{v^2(t)}{(1)}\, dt = \int_{-\infty}^{\infty} (1)\, i^2(t)\, dt$$

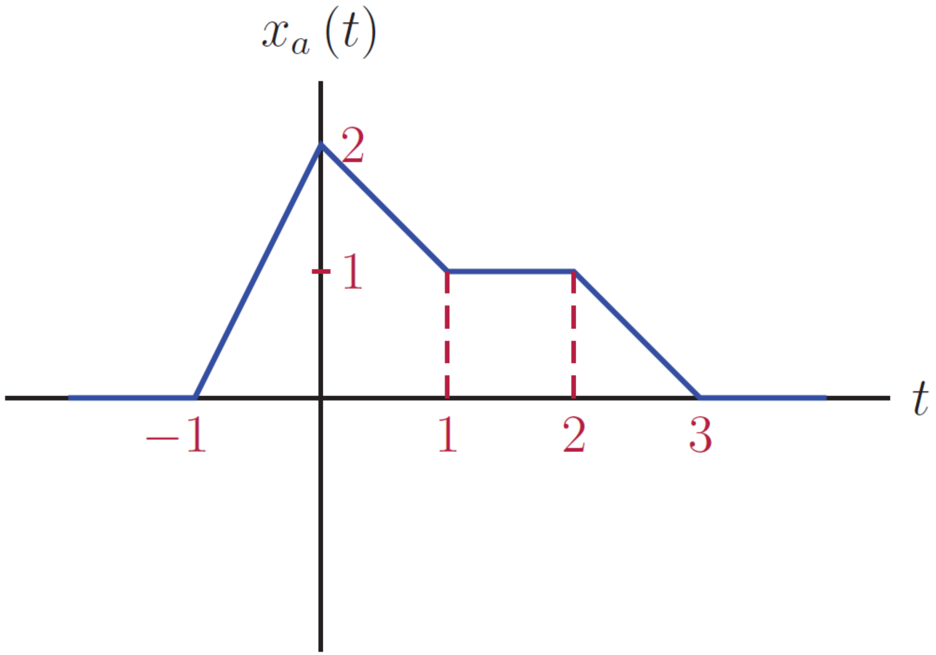
Problem 1.22 (a)

1.22. Determine the normalized energy of each of the signals shown in Fig. P.1.2.



Problem 1.22 (a) – Solution

$$x_a(t) = \begin{cases} 2t+2, & -1 < t < 0 \\ -t+2, & 0 < t < 1 \\ 1, & 1 < t < 2 \\ -t+3, & 2 < t < 3 \\ 0;, & \text{otherwise} \end{cases}$$



$$E_x = \int_{-1}^{0} (2t+2)^2\,dt + \int_{0}^{1} (-t+2)^2\,dt + \int_{1}^{2} (1)^2\,dt + \int_{2}^{3} (-t+3)^2\,dt = 5$$

Time Averaging Operator

- In preparation for defining the power in a signal, we need to first define the time average of a signal.
- We will use the operator $\langle \ldots \rangle$ to indicate time average.
- If the signal $x(t)$ is periodic with period T_0, its time average can be computed as

$$\langle x(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, dt$$

Time Averaging Operator: tavg_demo

Refer to: Pages 37 and 38, Eqns. (1.83) and (1.84), Example 1.8, Fig. 1.49.

$$\langle x(t) \rangle = \frac{1}{T_0} \int_0^{T_0} x(t)\, dt$$

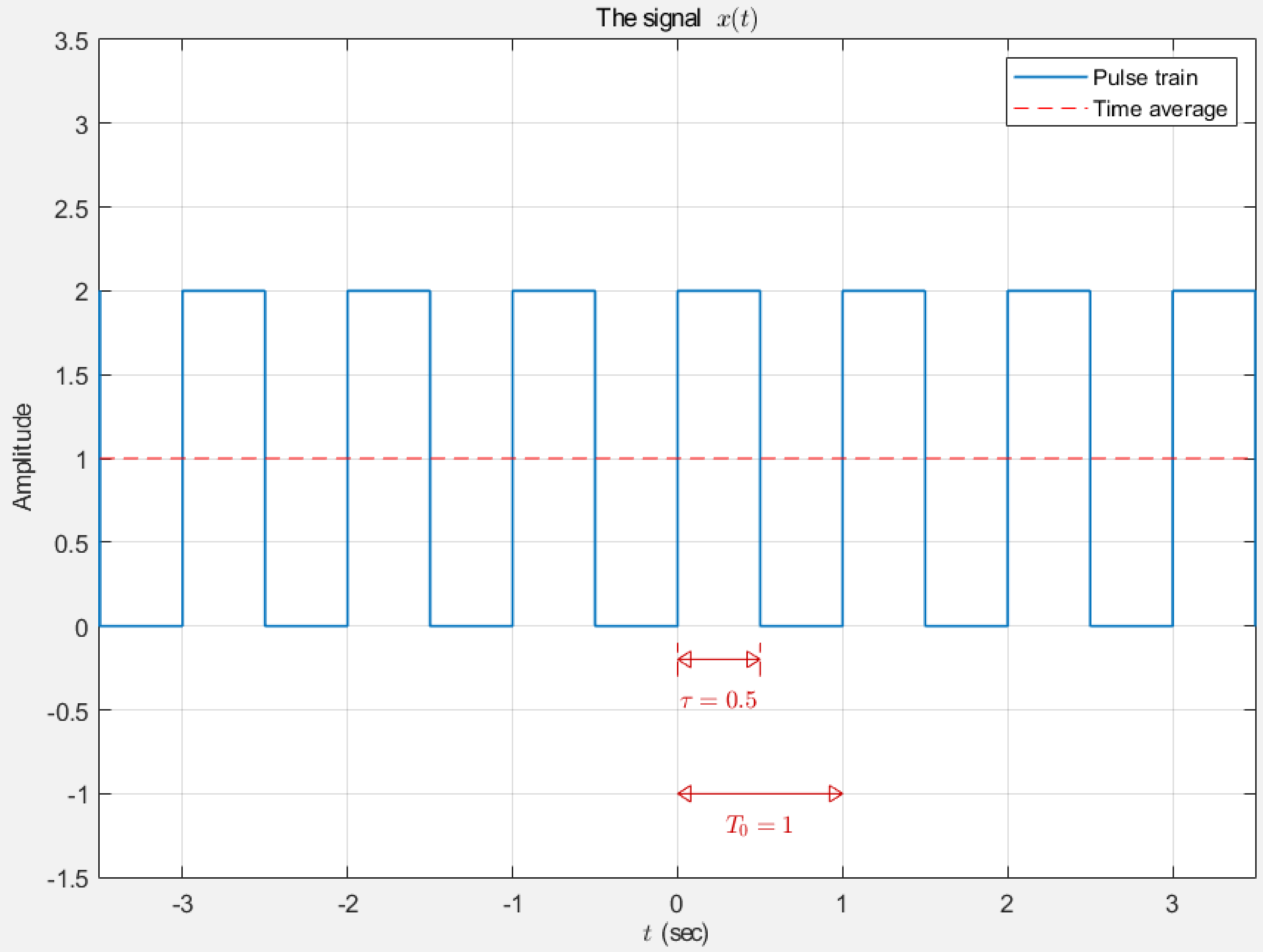
$$= A\, d = 1$$

Pulse amplitude (A): 2

Duty cycle (d): 0.5

Period (T0) in seconds: 1

☑ Display annotations



Power of a Signal

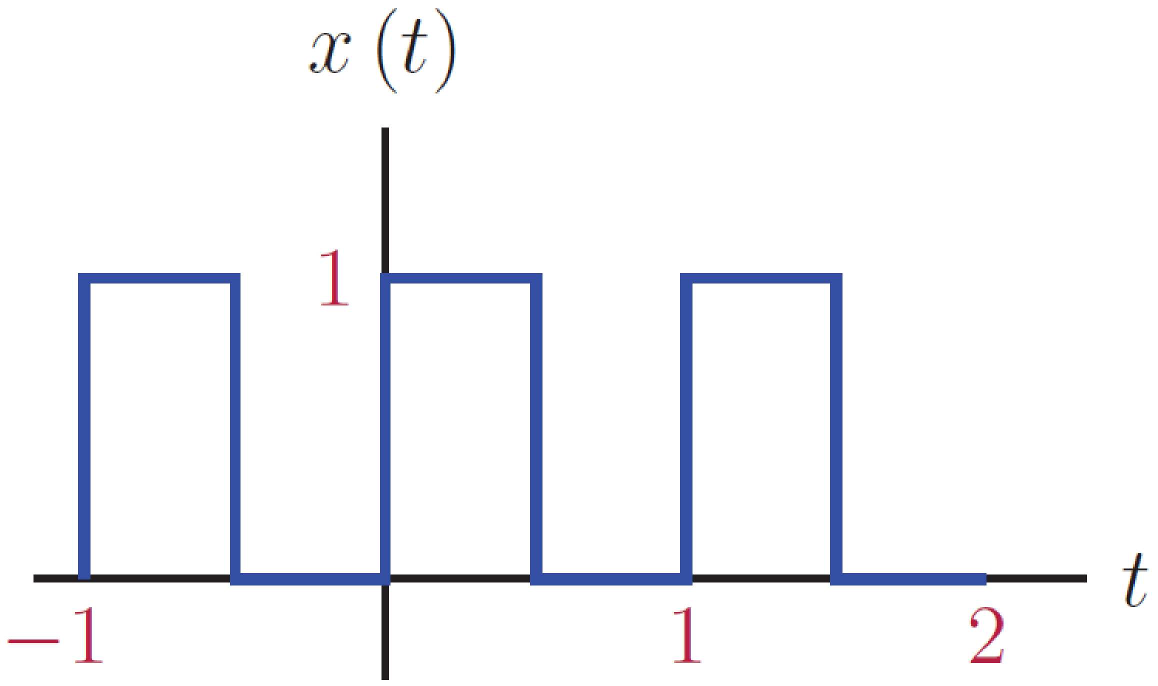
- For a periodic signal, the normalized average power defined as

$$P_x = \langle x^2(t) \rangle = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x^2(t)\, dt$$

- Energy signals are those that have finite energy and zero power.
- Power signals are those that have finite power and infinite energy.

Problem 1.23 (a)

1.23. Determine the normalized average power of each of the periodic signals shown in Fig. P.1.23.



Problem 1.23 (a) – Solution

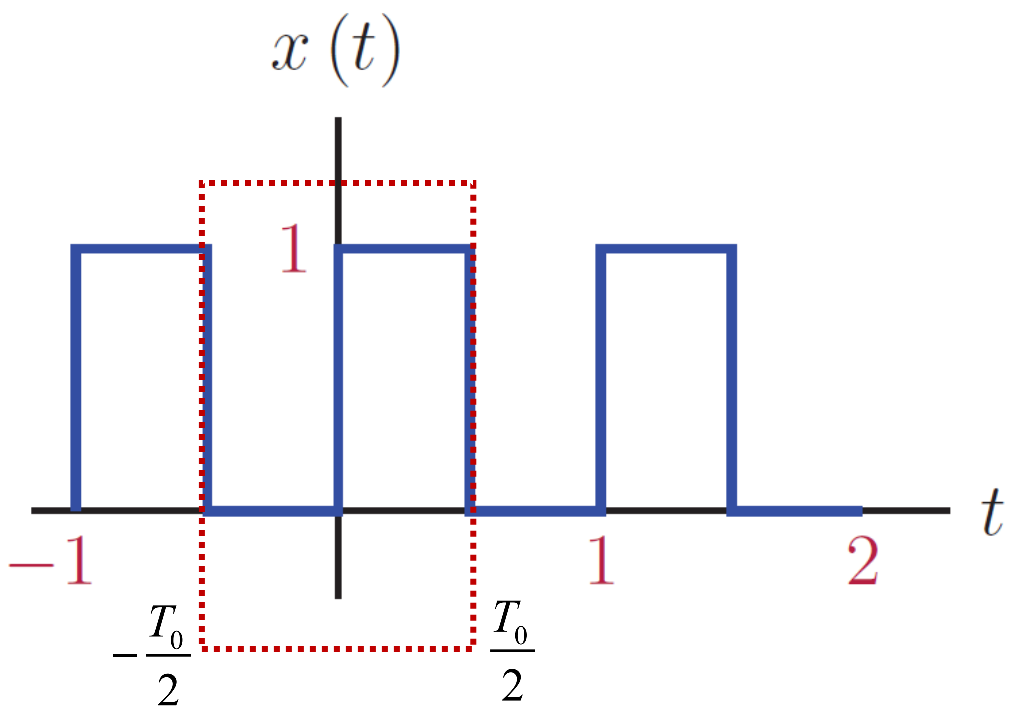
- The signal is not limited in time.

$$x(t) = \begin{cases} 0, & -0.5 < t < 0 \\ 1, & 0 < t < 0.5 \end{cases}$$

$$x(t+k) = x(t)$$

for all t, and all integers k

- The period of the signal is $T_0 = 1$



$$P_x = \left\langle |x(t)|^2 \right\rangle = \frac{1}{1} \int_{-0.5}^{0.5} x^2(t)\,dt = \int_{-0.5}^{0} (0)^2\,dt + \int_{0}^{0.5} (1)^2\,dt = t\Big|_0^{0.5} = 0.5$$

Problem 1.23 (a) – Another Solution

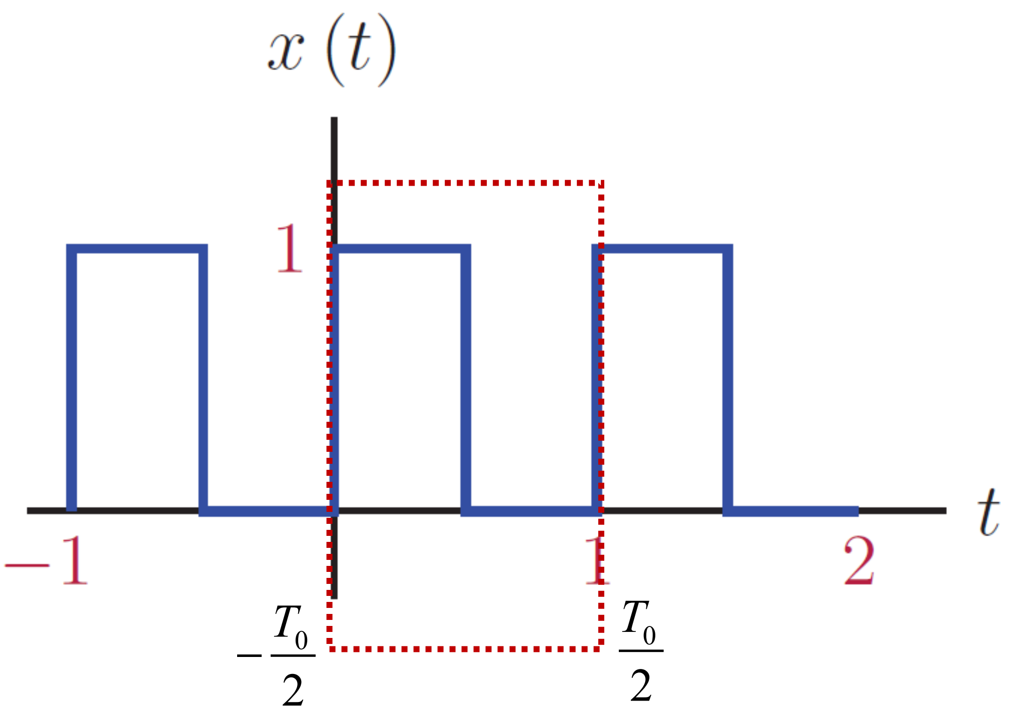
- The signal is not limited in time.

$$x(t) = \begin{cases} 1, & 0 < t < 0.5 \\ 0, & 0.5 < t < 1 \end{cases}$$

$$x(t+k) = x(t)$$

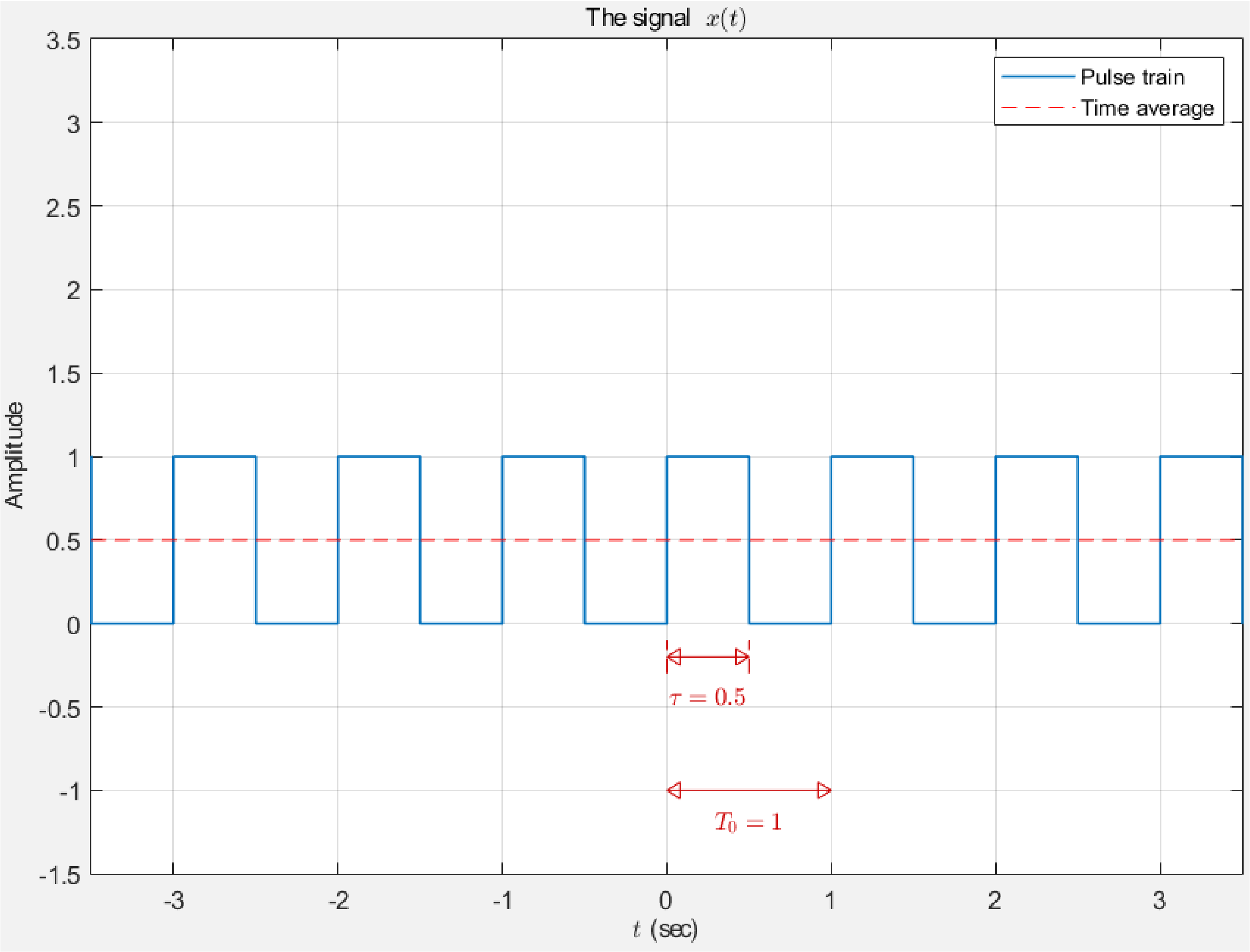
for all t, and all integers k

- The period of the signal is $T_0 = 1$



$$P_x = \left\langle |x(t)|^2 \right\rangle = \frac{1}{1}\int_0^1 x^2(t)\,dt = \int_0^{0.5} (1)^2\,dt + \int_{0.5}^{1} (0)^2\,dt = t\Big|_0^{0.5} = 0.5$$

Problem 1.23 (a) – Justification

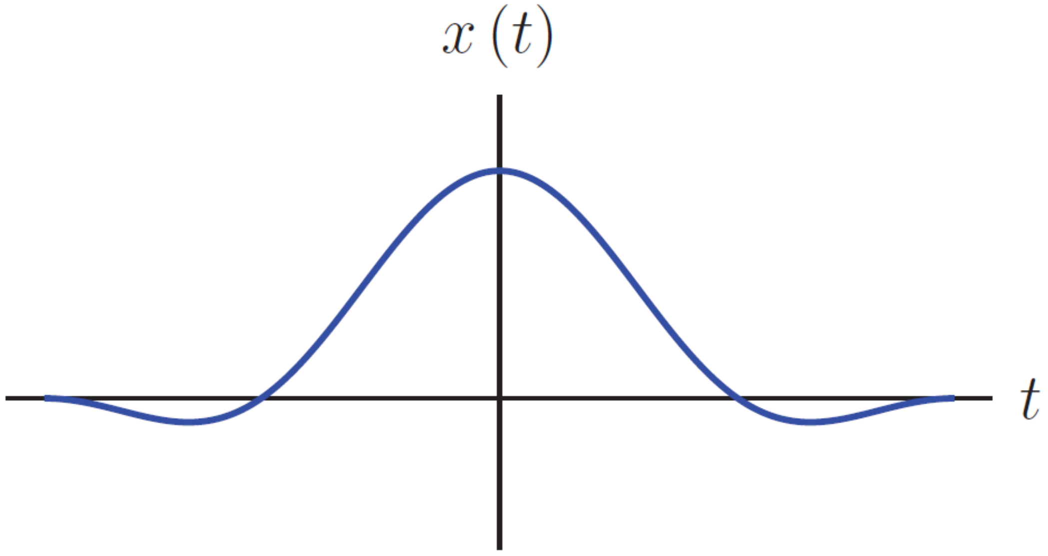


Even and Odd Symmetry

- Some signals have certain symmetry properties that could be utilized in a variety of ways in the analysis.
- A real-valued signal is said to have even symmetry if it has the property

$$x(-t) = x(t)$$

- A signal with even symmetry remains unchanged when it is time reversed.

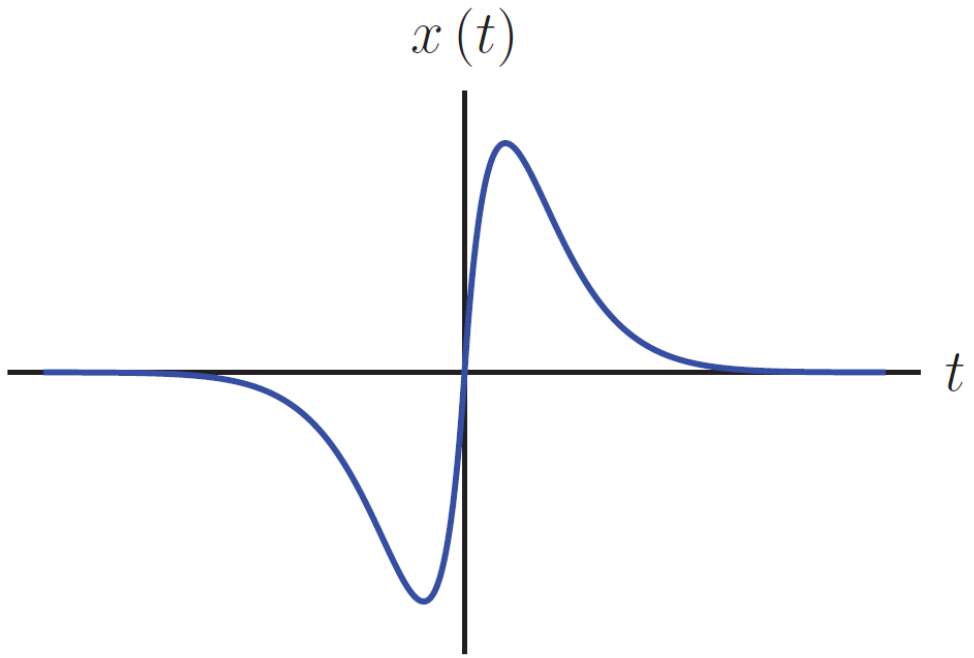


Even and Odd Symmetry

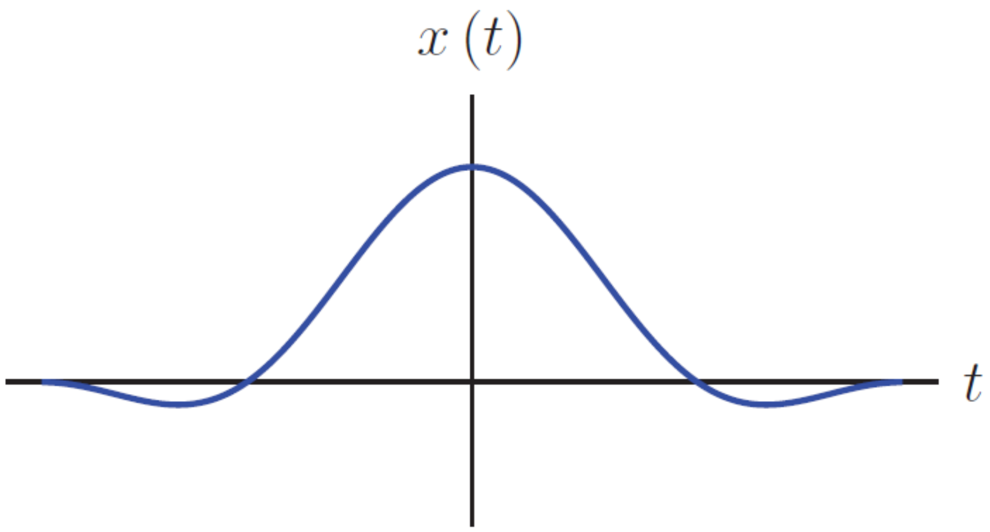
- A real-valued signal is said to have odd symmetry if it has the property

$$x(-t) = -x(t)$$

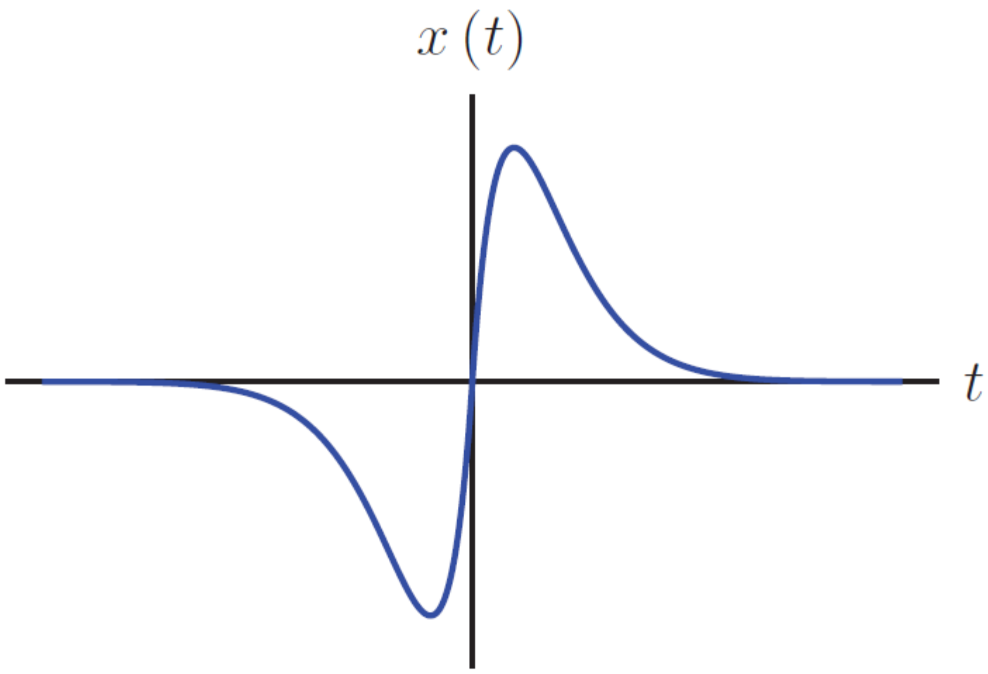
- Time reversal has the same effect as negation on a signal with odd symmetry.



Even and Odd Symmetry



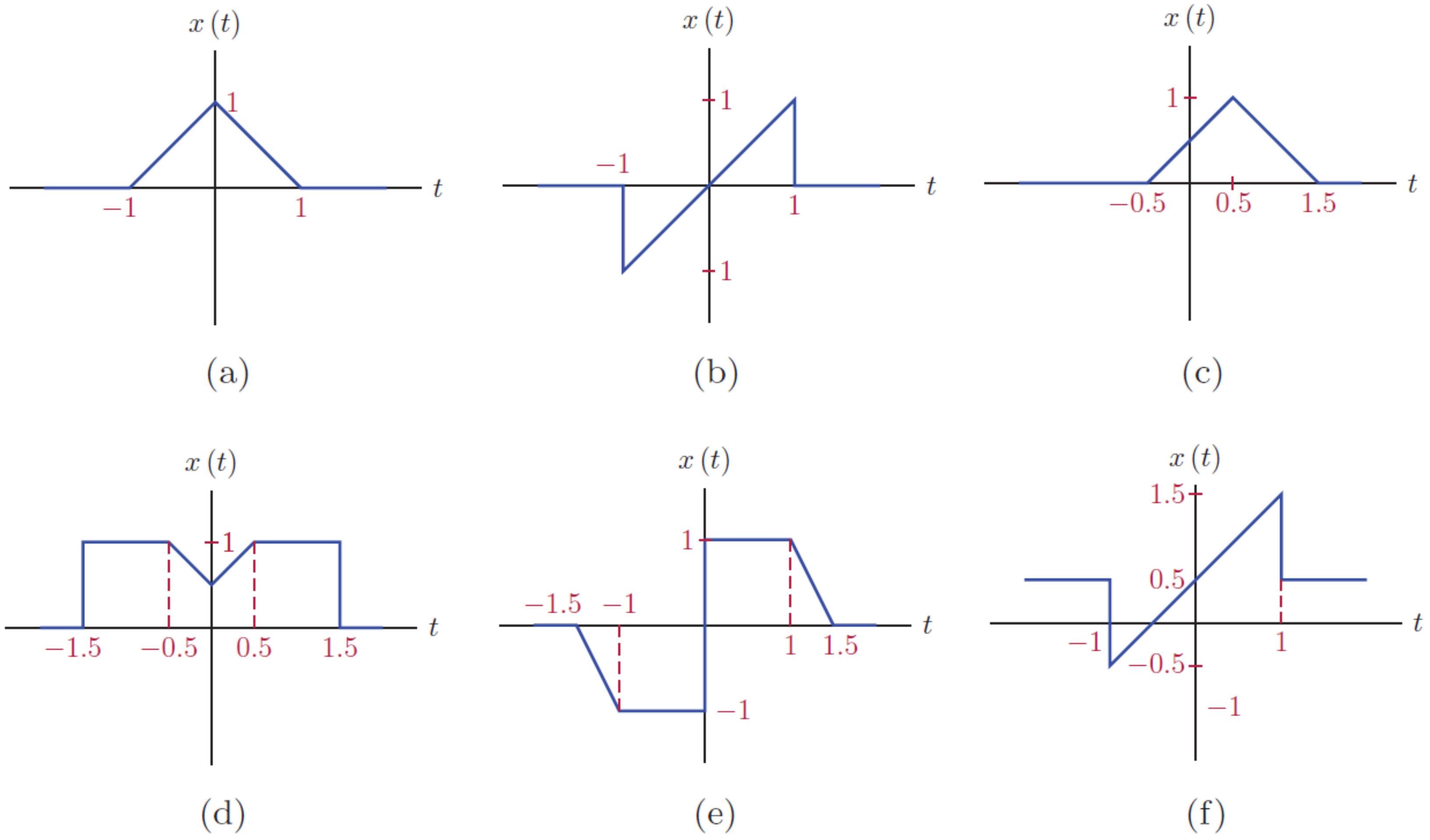
Even signal



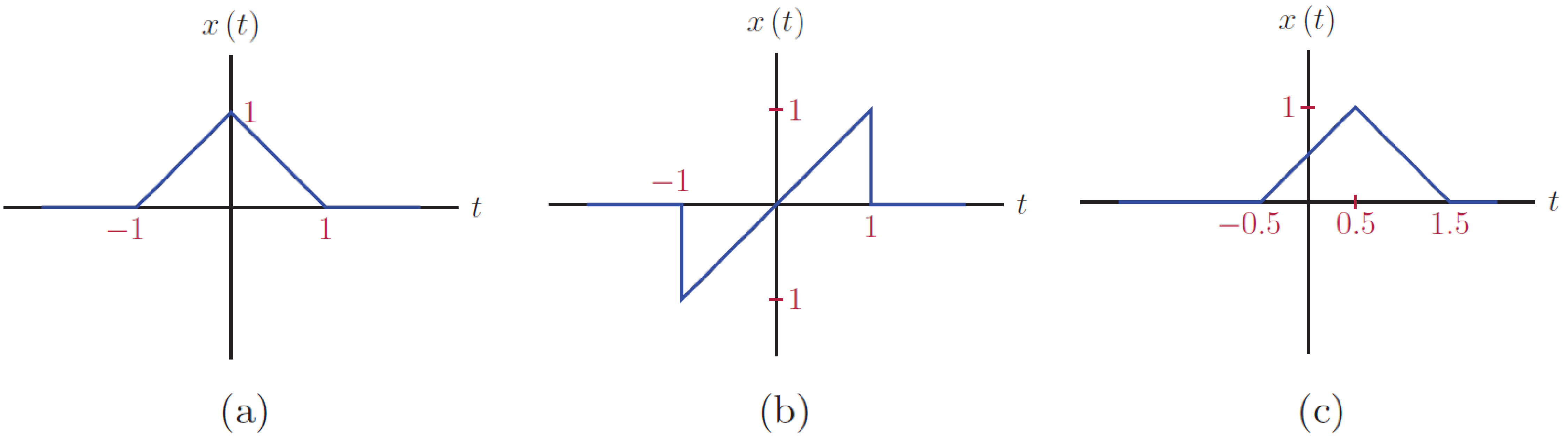
odd signal

Problem 1.25

1.25. Identify which of the signals in Fig. P.1.25 are even, which ones are odd, and which signals are neither even nor odd.



Problem 1.25 – Solution

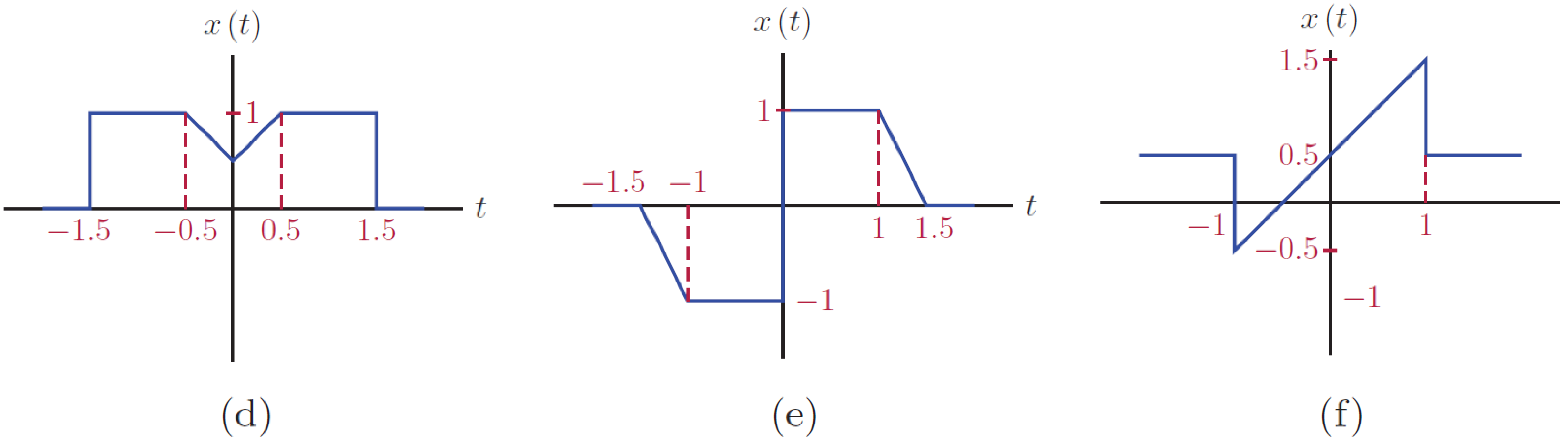


a. Even

b. Odd

c. Neither even nor odd

Problem 1.25 – Solution



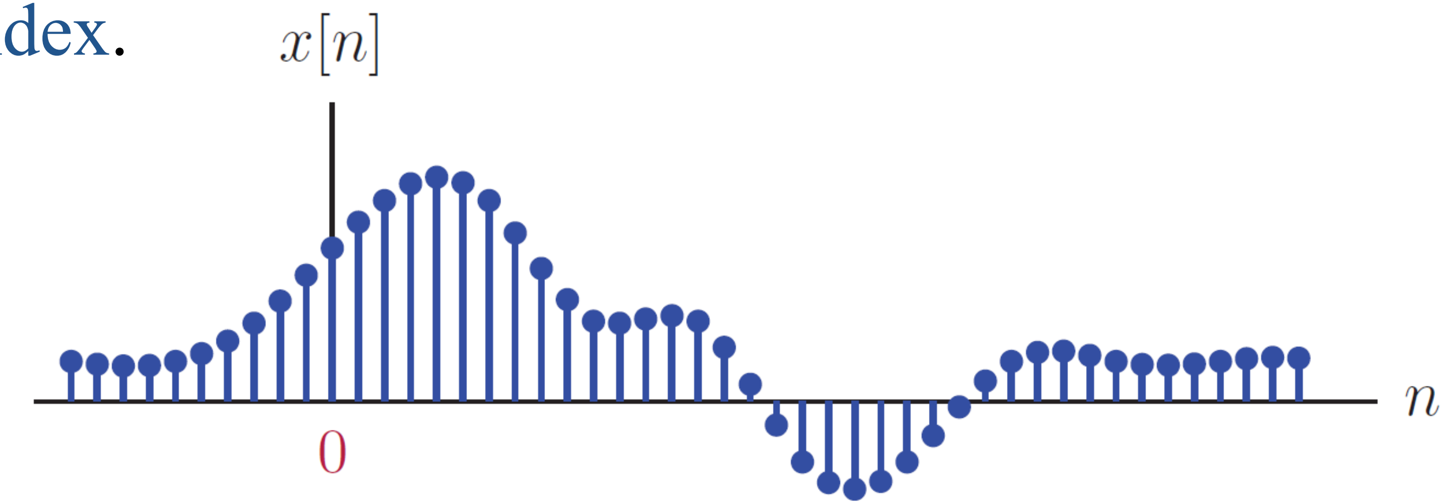
d. Even

e. Odd

f. Neither even nor odd

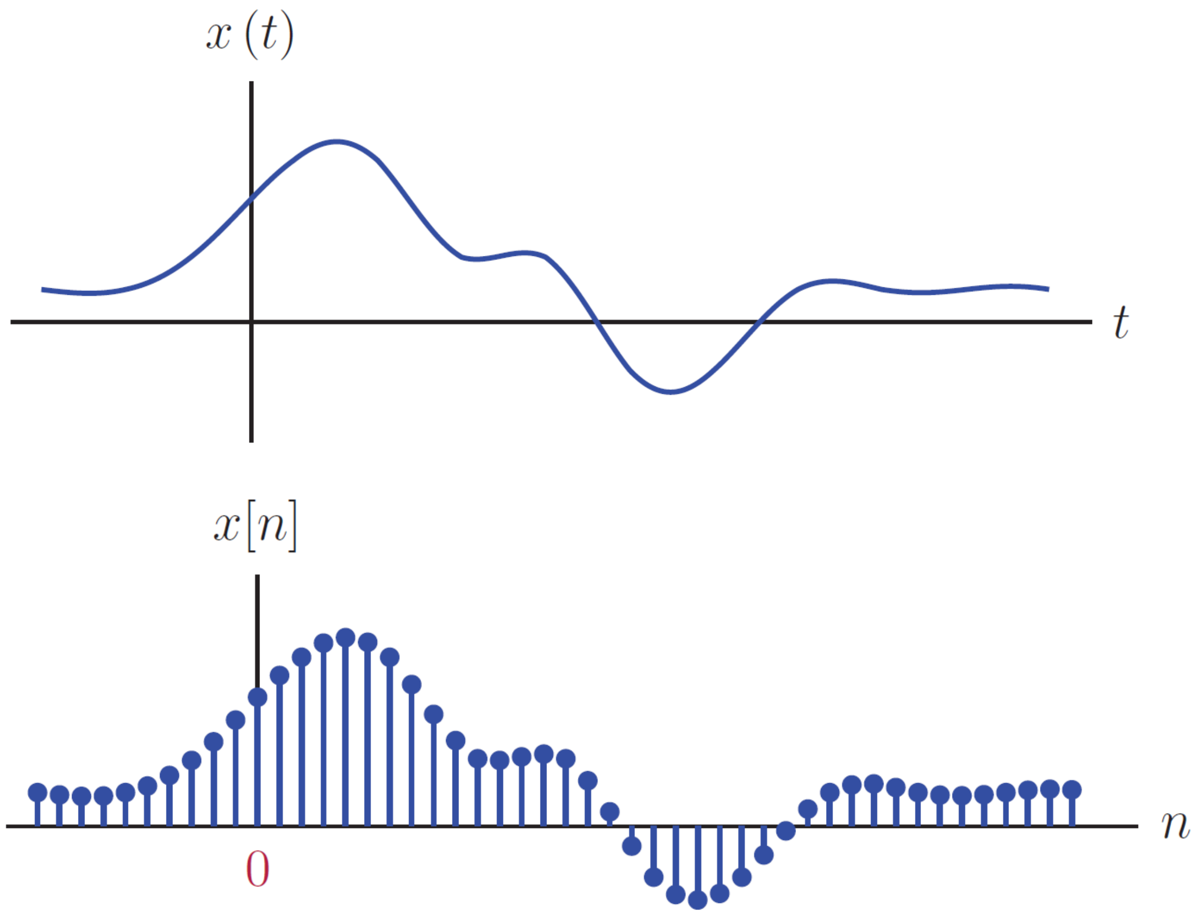
Discrete-Time Signals

- Discrete-time signals are not defined at all time instants.
- Instead, they are defined only at time instants that are integer multiples of a fixed time increment T, that is, at $t = n$T.
- The mathematical model for a discrete-time signal is a function $x[n]$ in which independent variable n is an integer, and is referred to as the sample index.



Signal Operations

- Arithmetic operations for discrete-time signals bear strong resemblance to their continuous-time counterparts.

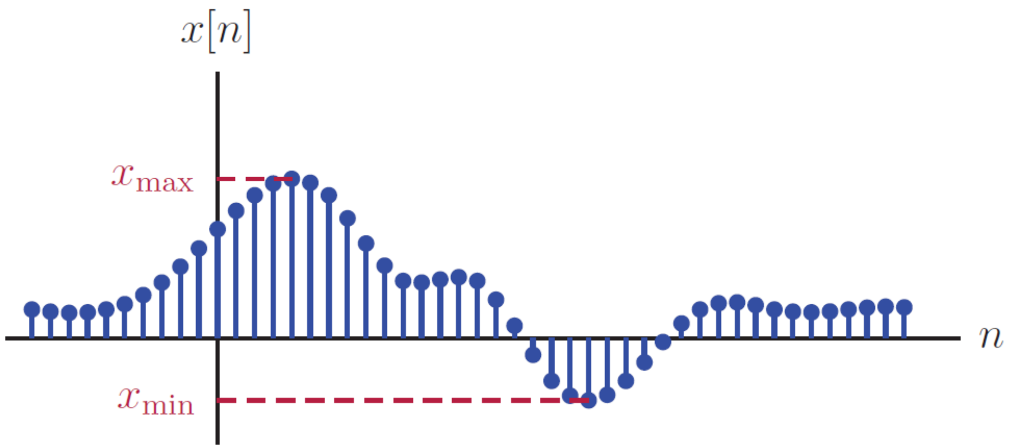


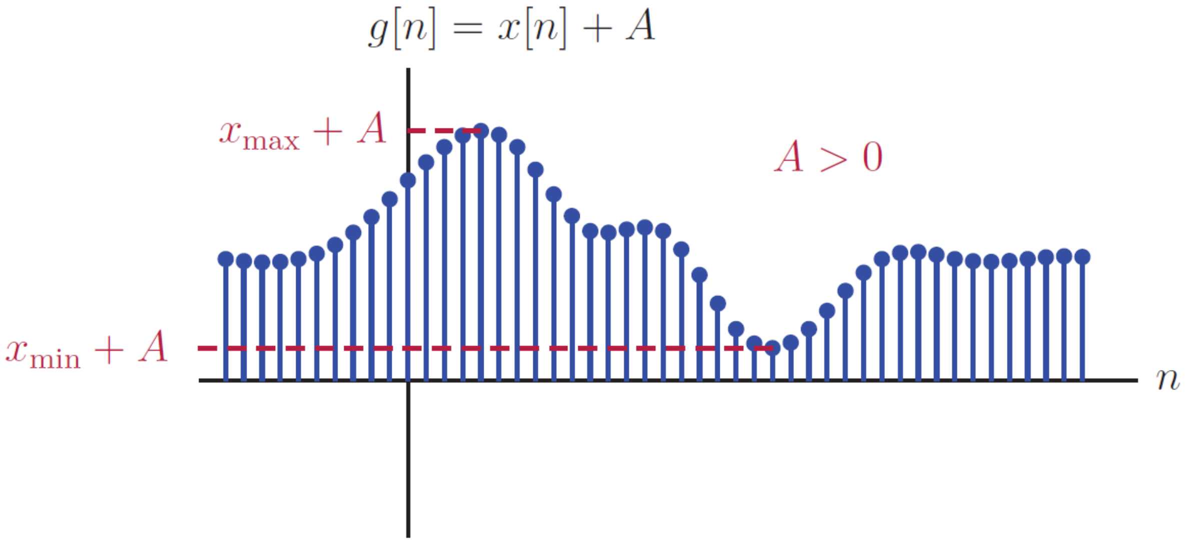
Arithmetic Operations: Addition of a Constant Offset

- A constant offset value can be added to this signal to obtain

$$g[n] = x[n] + A$$

- The offset A is added to each sample of the signal $x[n]$.





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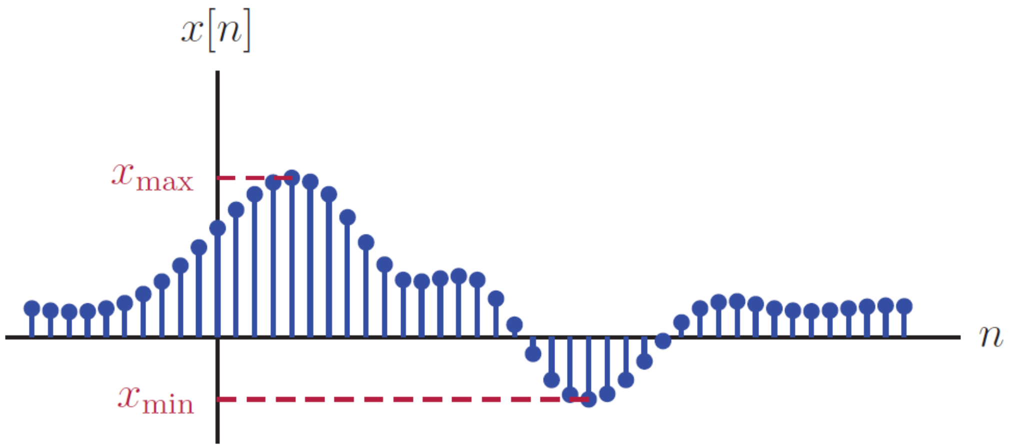
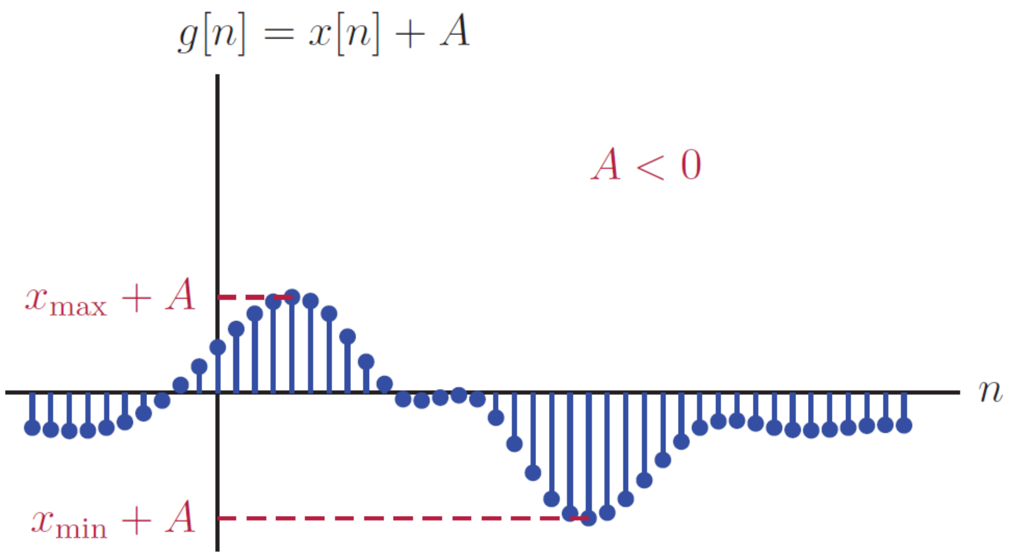


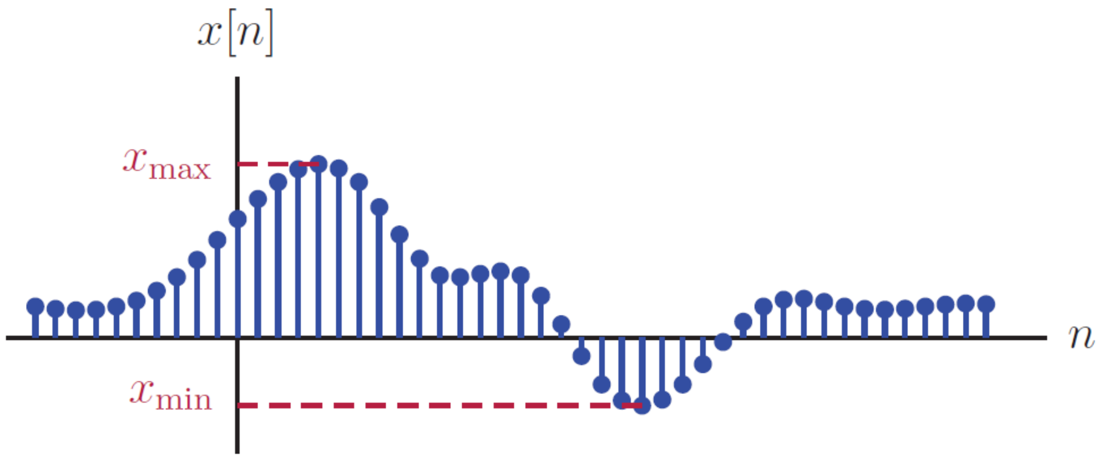


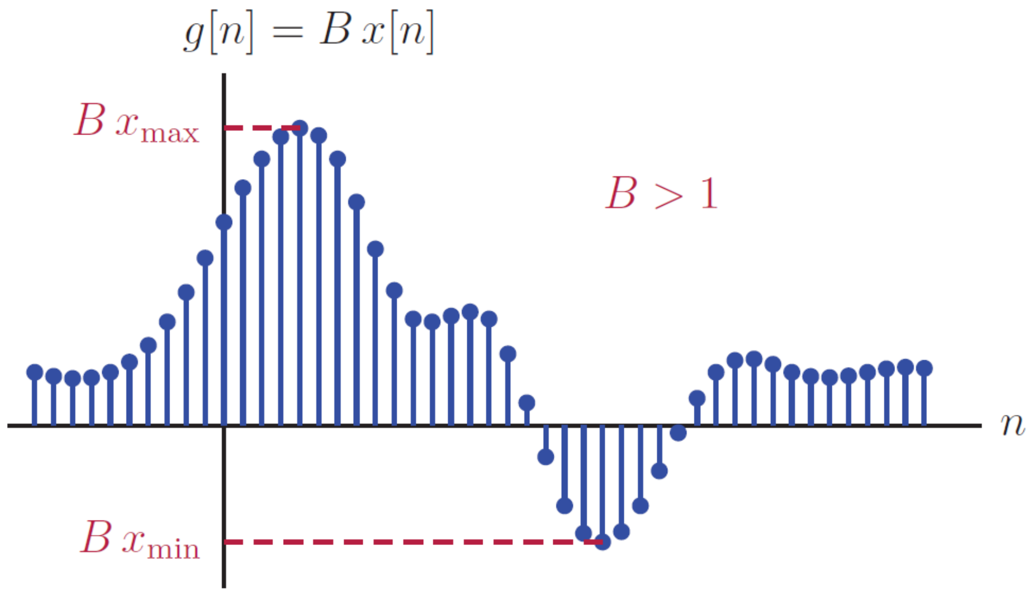
Arithmetic Operations: Multiplication By a Constant Gain Factor

- Multiplication of the signal $x[n]$ with gain factor B is expressed as

$$g[n] = Bx[n]$$

- The value of each sample of the signal $g[n]$ is equal to the product of the corresponding sample of $x[n]$ and the constant gain factor B.





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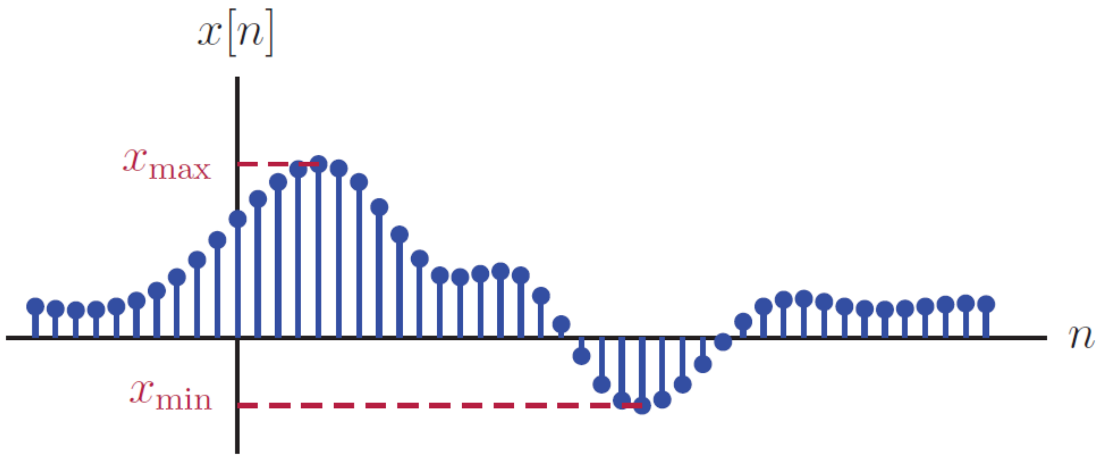
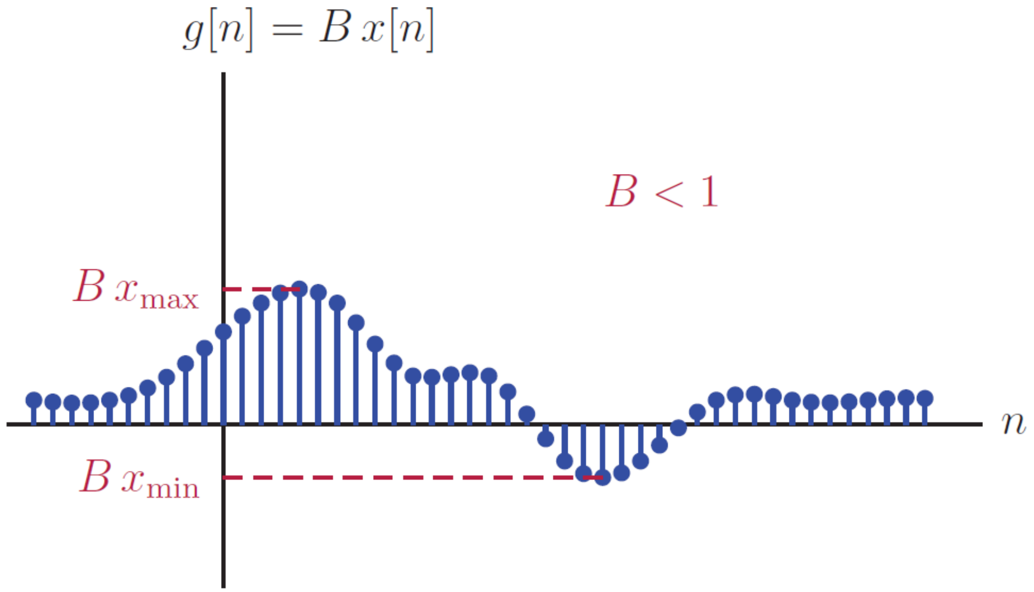
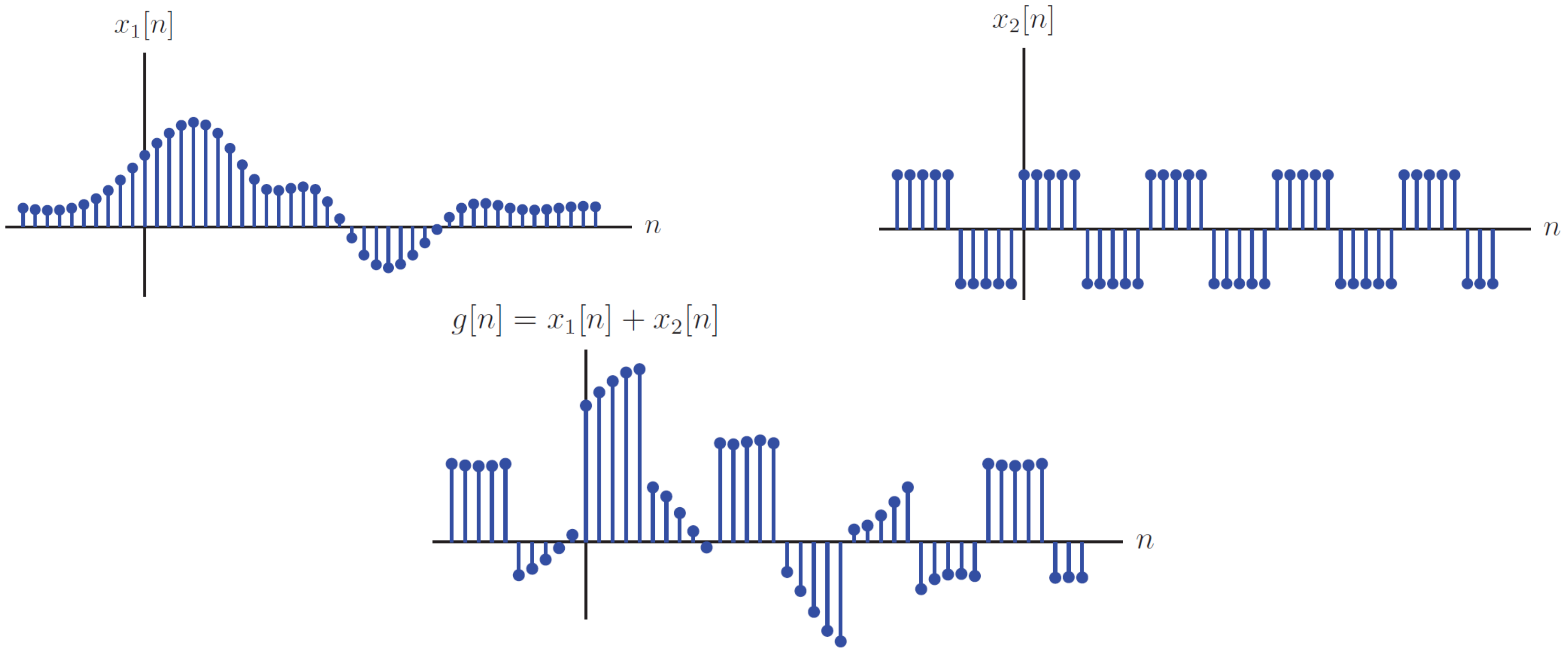




Arithmetic Operations: Adding Signals

- Addition of two discrete-time signals is accomplished by adding the amplitudes of the corresponding samples of the two signals.

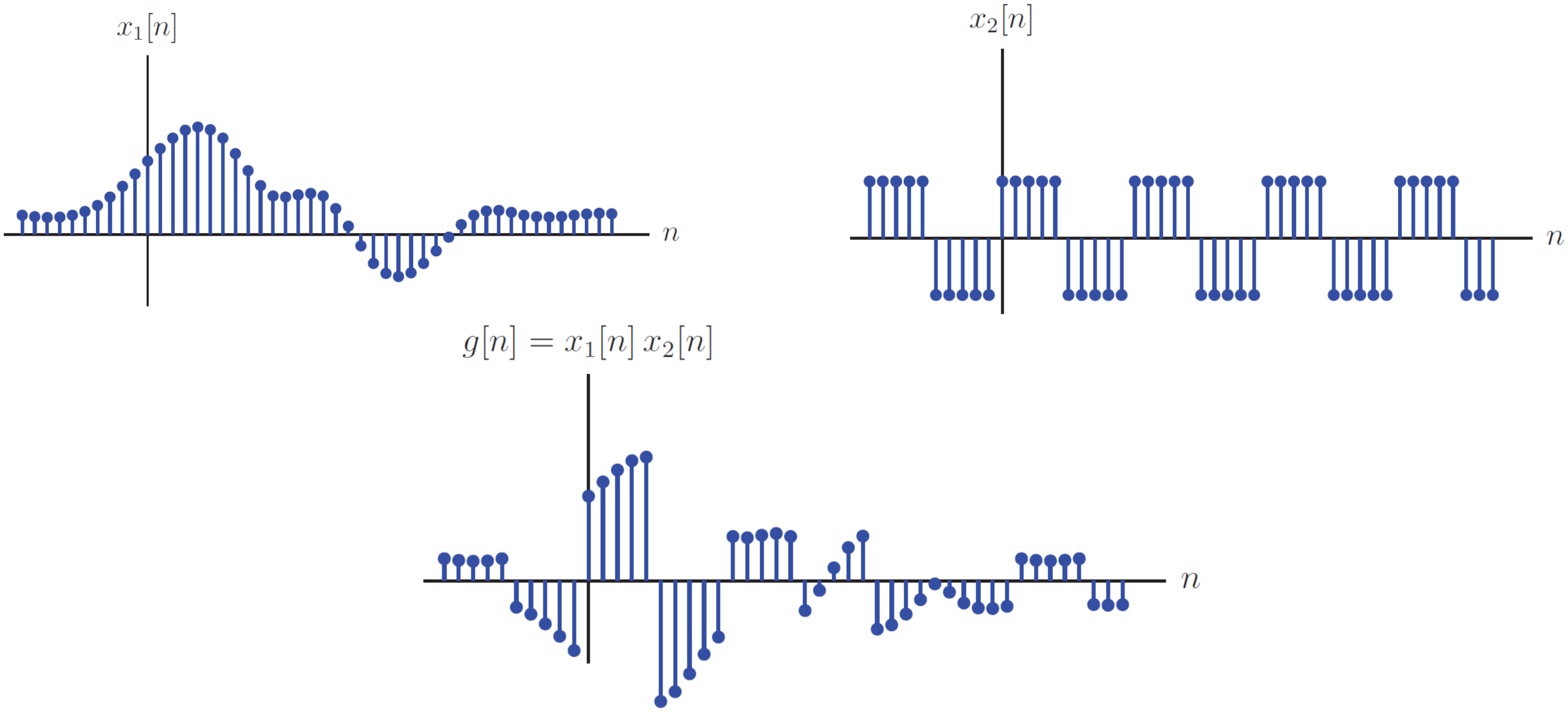
$$g[n] = x_1[n] + x_2[n]$$



Arithmetic Operations: Multiplying Signals

- Two discrete-time signals can also be multiplied in a similar manner.

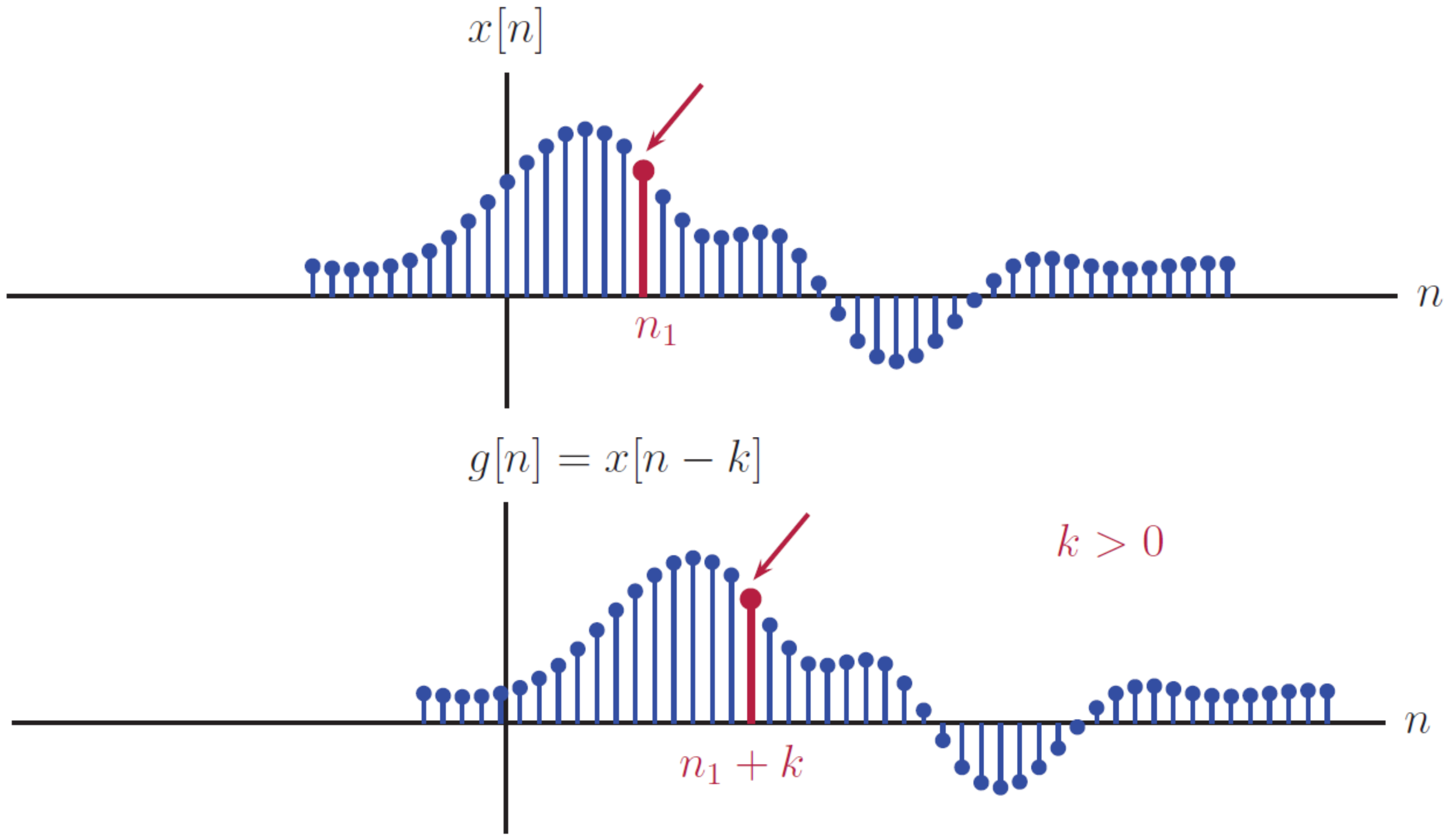
$$g[n] = x_1[n]\, x_2[n]$$



Arithmetic Operations: Time Shifting

- Time shifting operations must utilize integer shift parameters (k).

$$g[n] = x[n-k]$$



Arithmetic Operations: Time Shifting

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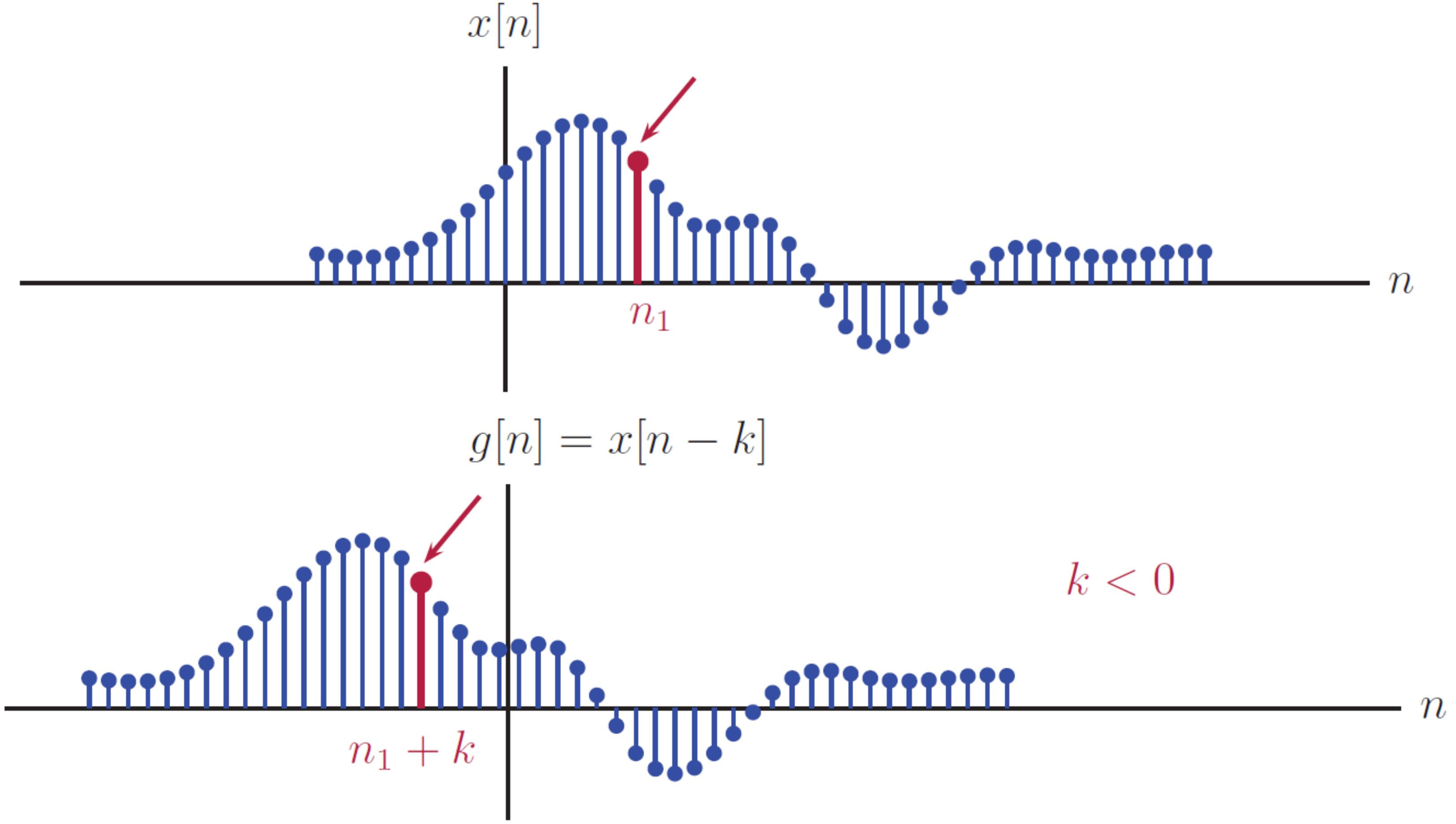


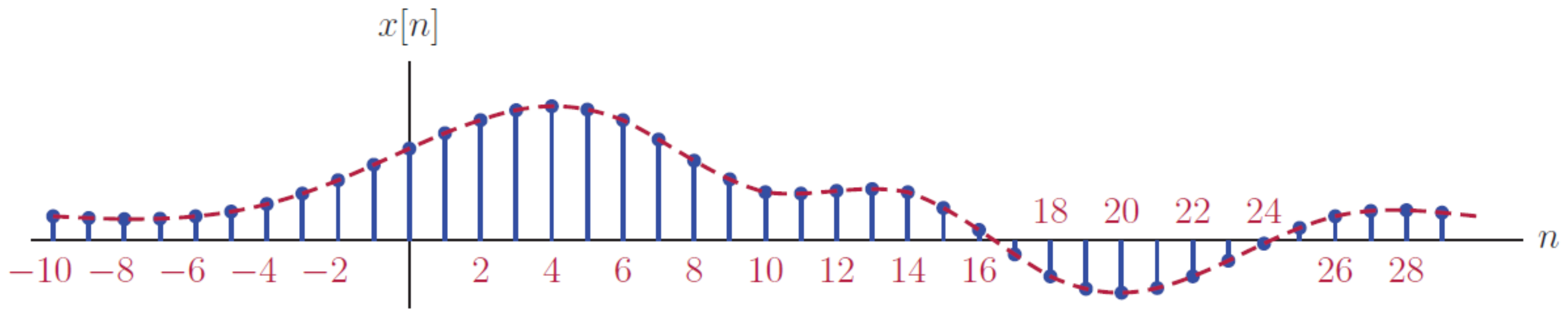
Arithmetic Operations: Time Scaling (Downsampling)

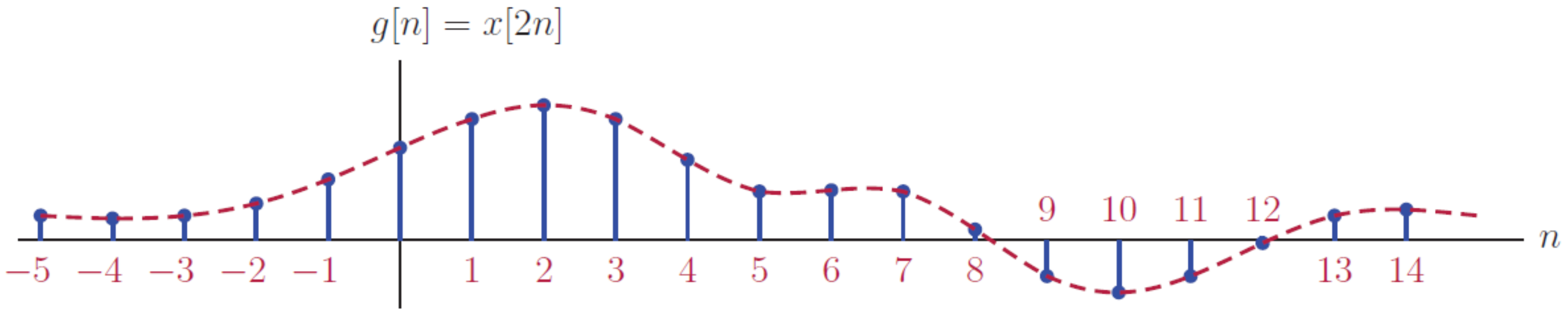
- A downsampled version of the signal $x[n]$ is obtained through

$$g[n] = x[kn], \quad k\text{: integer}$$

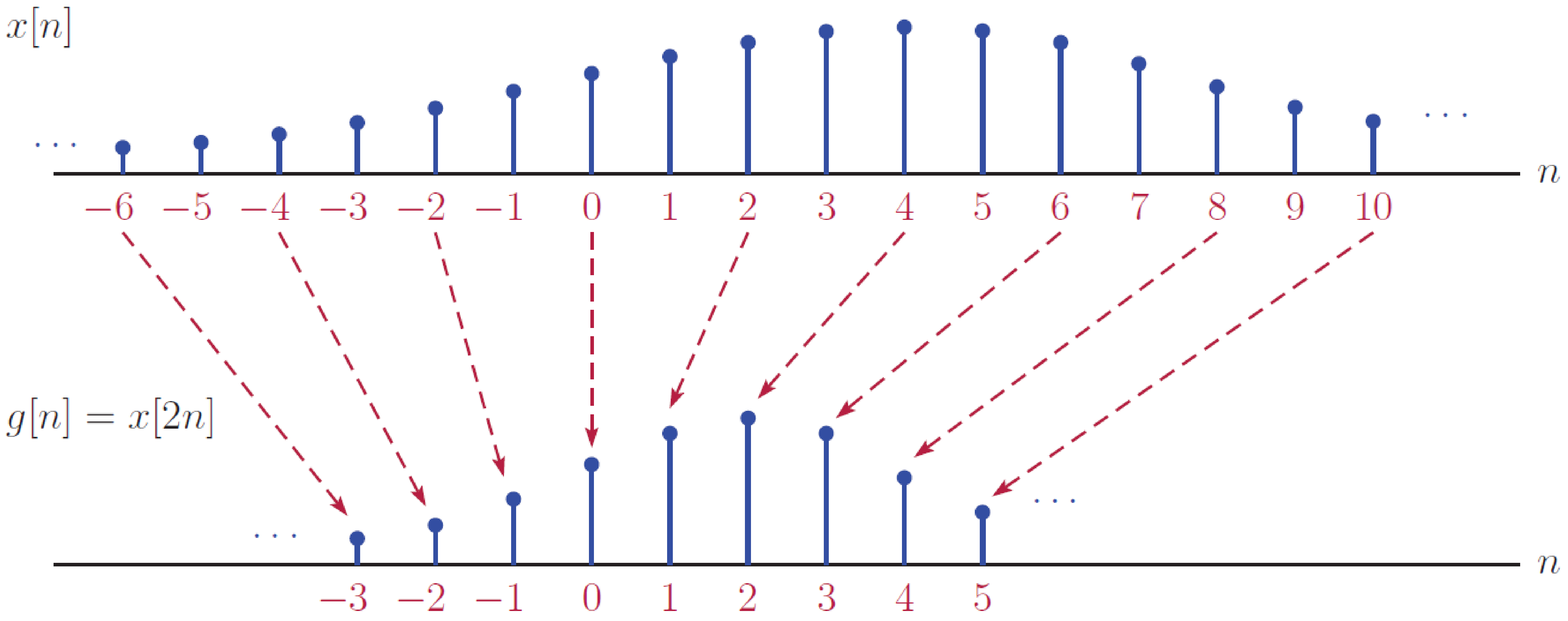
- For $k = 2$, we have

$$g[-1] = x[-2], \quad g[0] = x[0], \quad g[1] = x[2], \quad g[2] = x[4], \quad \ldots$$





Arithmetic Operations: Time Scaling (Downsampling)



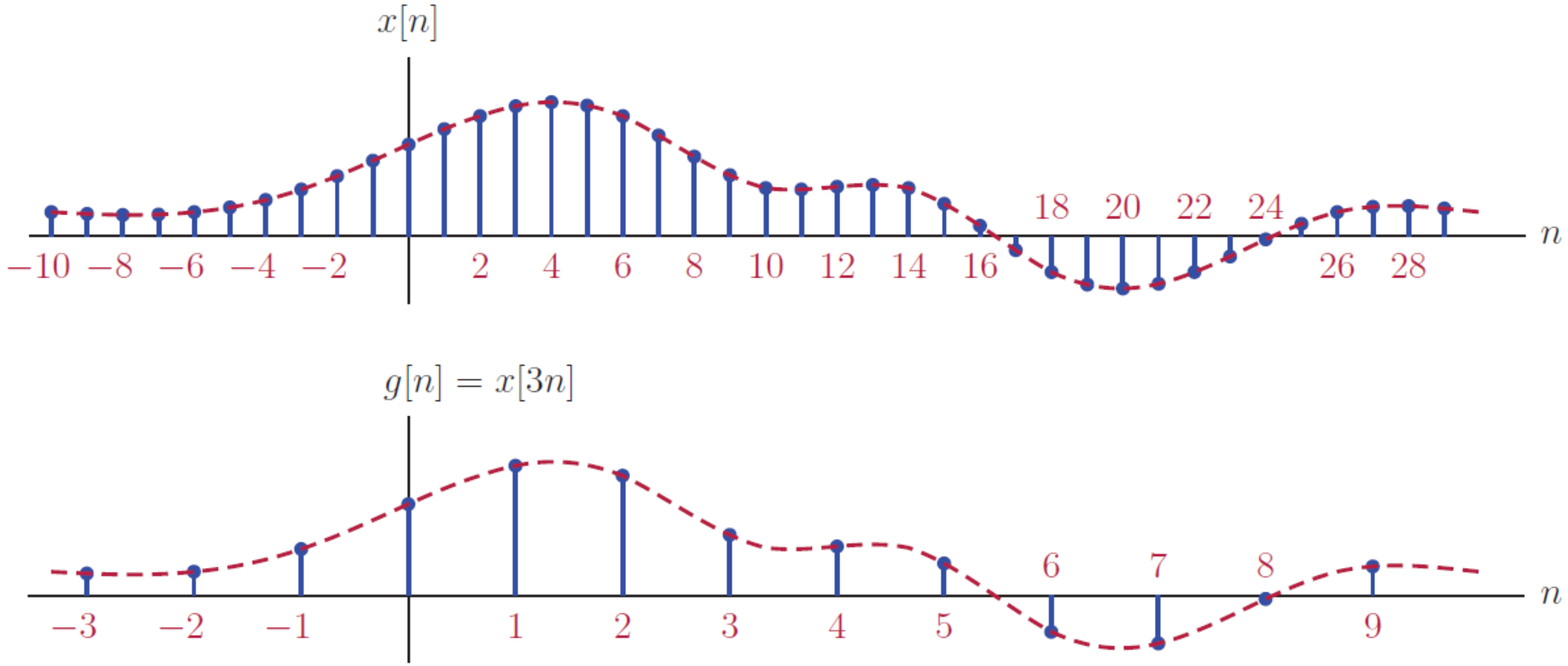
Arithmetic Operations: Time Scaling (Downsampling)

- A downsampled version of the signal $x[n]$ is obtained through

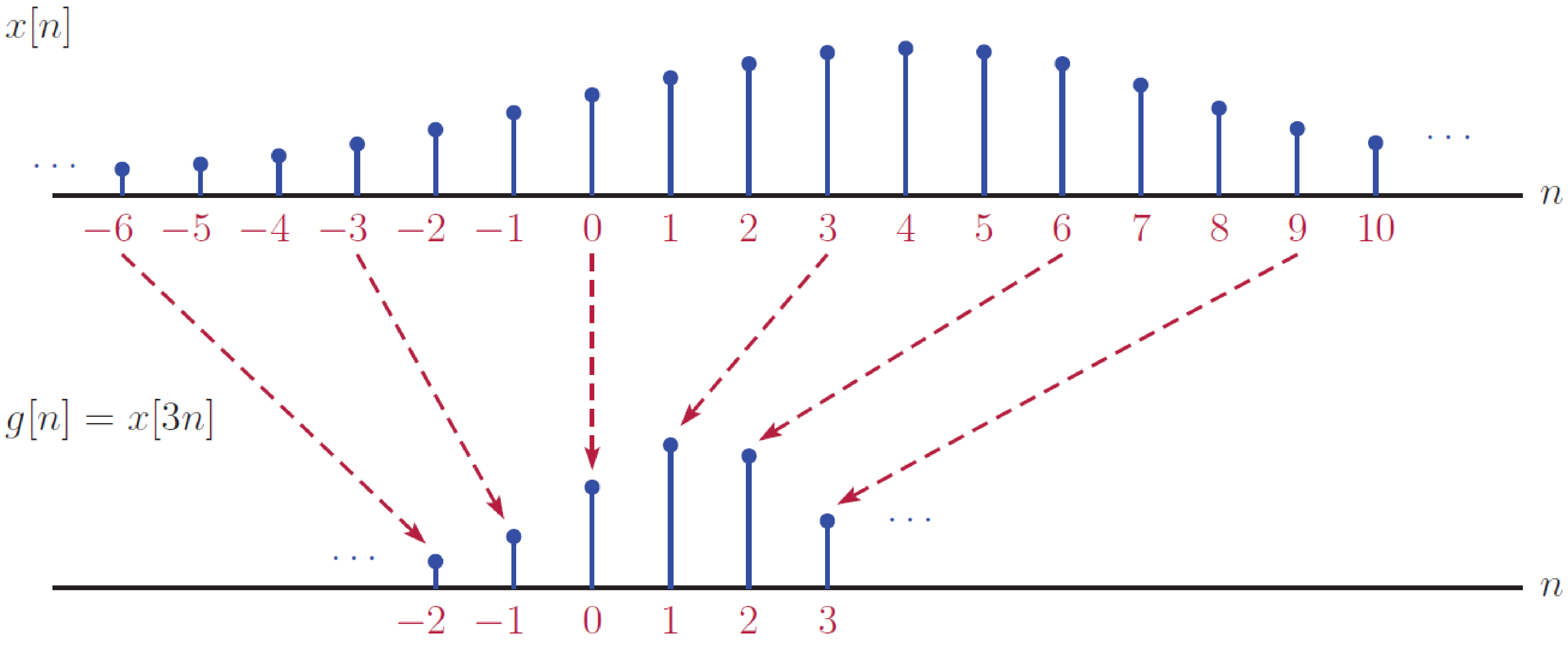
$$g[n] = x[kn], \quad k\text{: integer}$$

- For $k = 3$, we have

$$g[-1] = x[-3], \quad g[0] = x[0], \quad g[1] = x[3], \quad g[2] = x[6], \quad \ldots$$



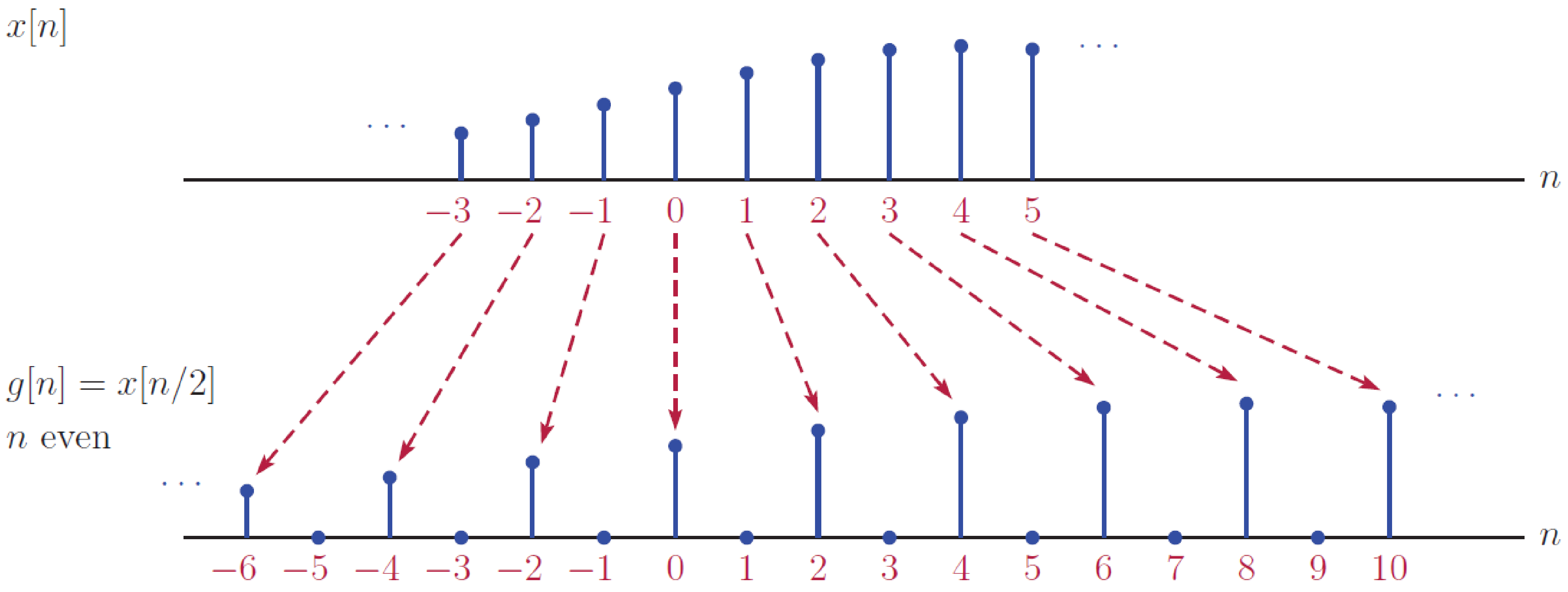
Arithmetic Operations: Time Scaling (Downsampling)



Arithmetic Operations: Time Scaling (Upsampling)

- A upsampled version of the signal $x[n]$ is obtained through

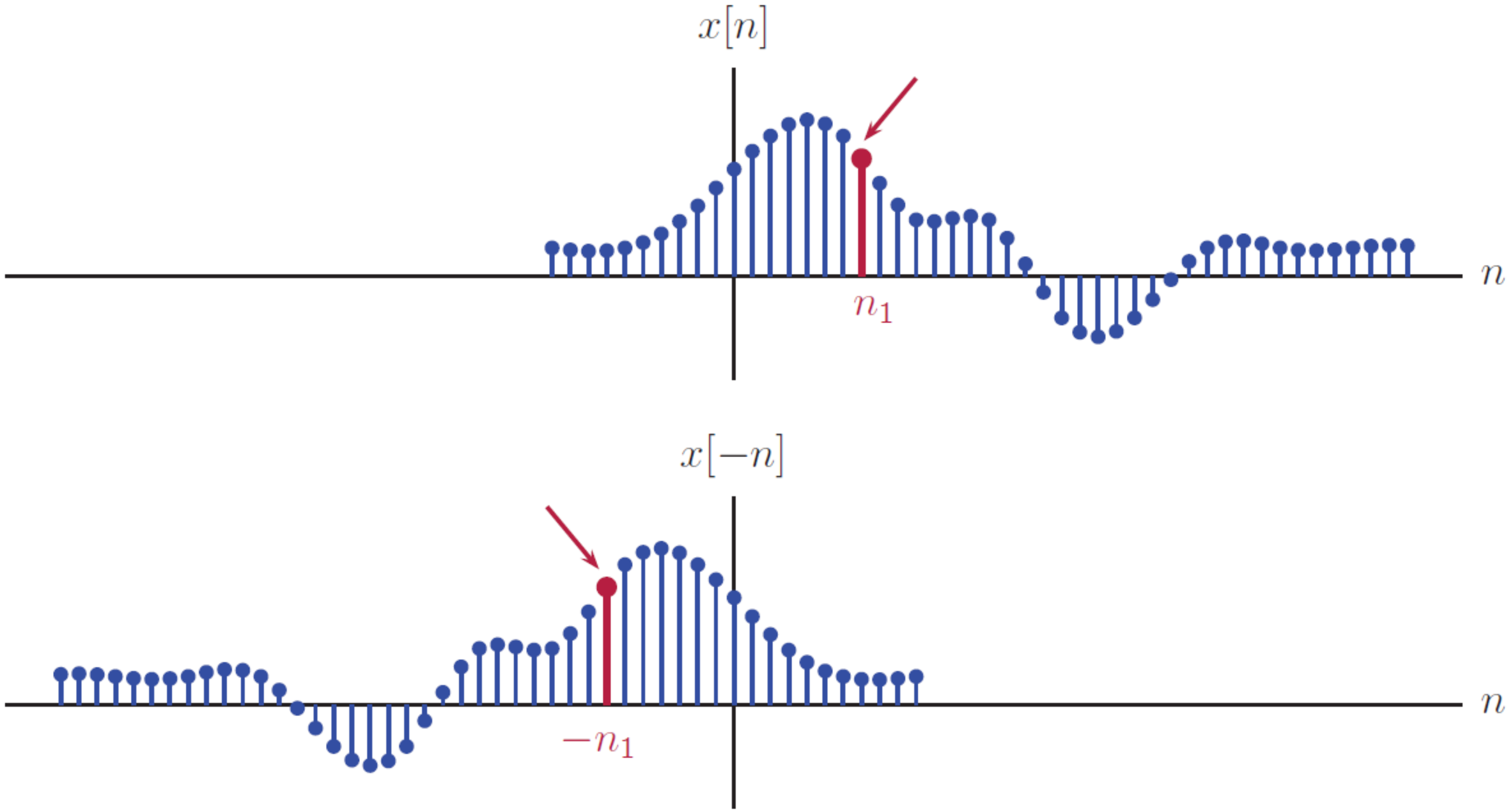
$$g[n] = \begin{cases} x[n/k] & \text{if } n/k \text{ is integer} \\ 0\,, & \text{otherwise} \end{cases}$$



Arithmetic Operations: Time Reversal

- A time reversed version of the signal x[n] is

$$g[n] = x[-n]$$

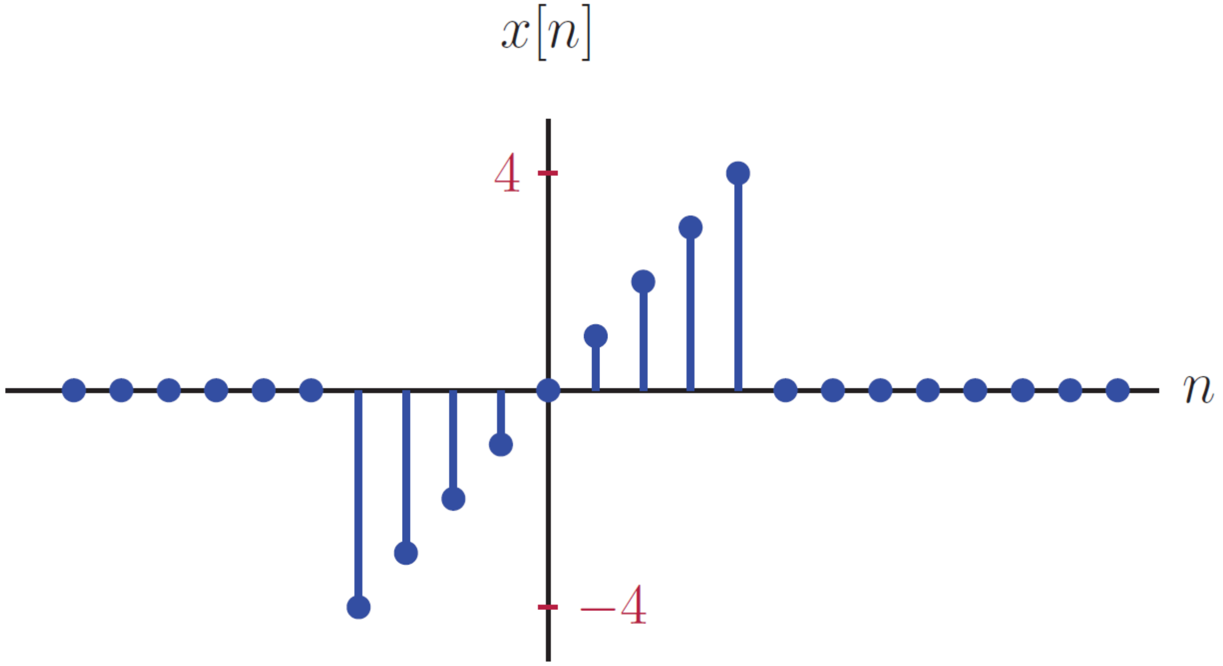


Problem 1.33

1.33. For the signal $x[n]$ shown in Fig. P.1.33, sketch the following signals.

a. $g[n] = x[n-3]$

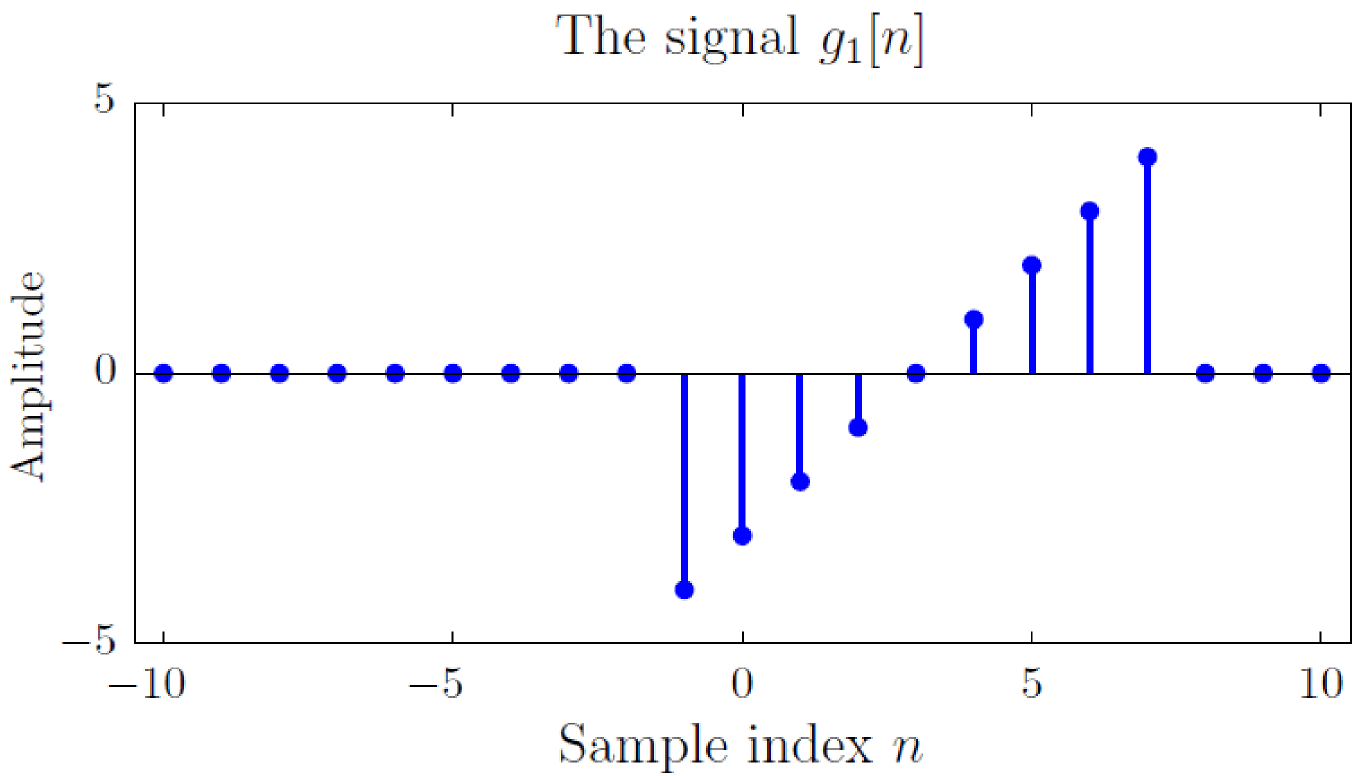
e. $g[n] = \begin{cases} x[n/2], & \text{if } n/2 \text{ is integer} \\ 0, & \text{otherwise} \end{cases}$



Problem 1.33 (a) – Solution

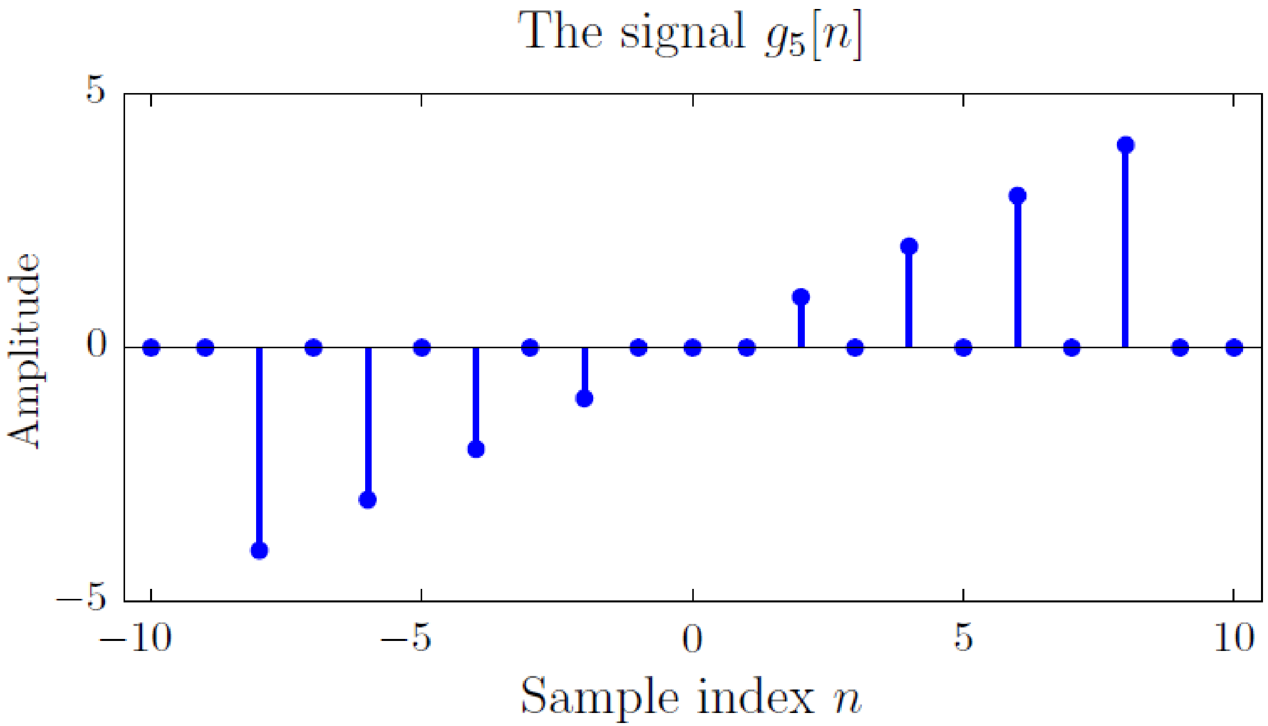
Time shifting

$$g_1[n] = x[n-3]$$



Problem 1.33 (e) – Solution

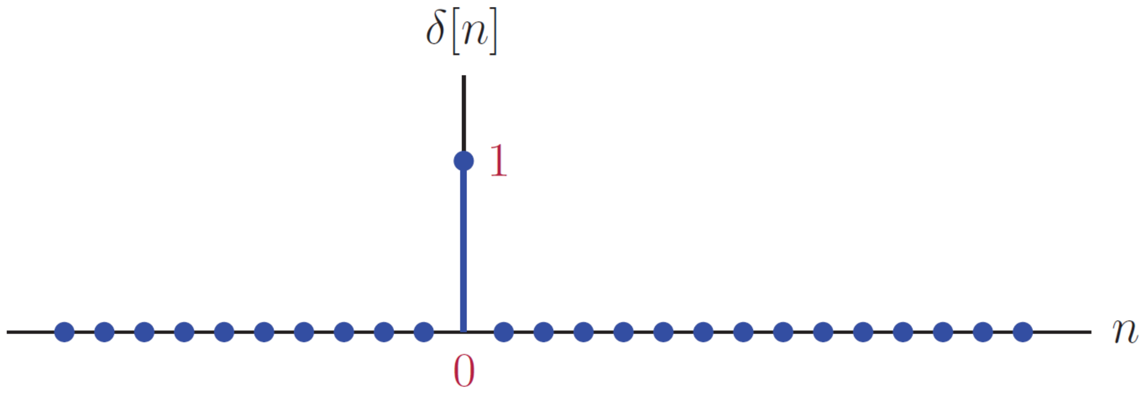
$$g_5[n] = \begin{cases} x[n/2], & \text{if } n/2 \text{ is integer} \\ 0, & \text{otherwise} \end{cases}$$



Unit-Impulse Function

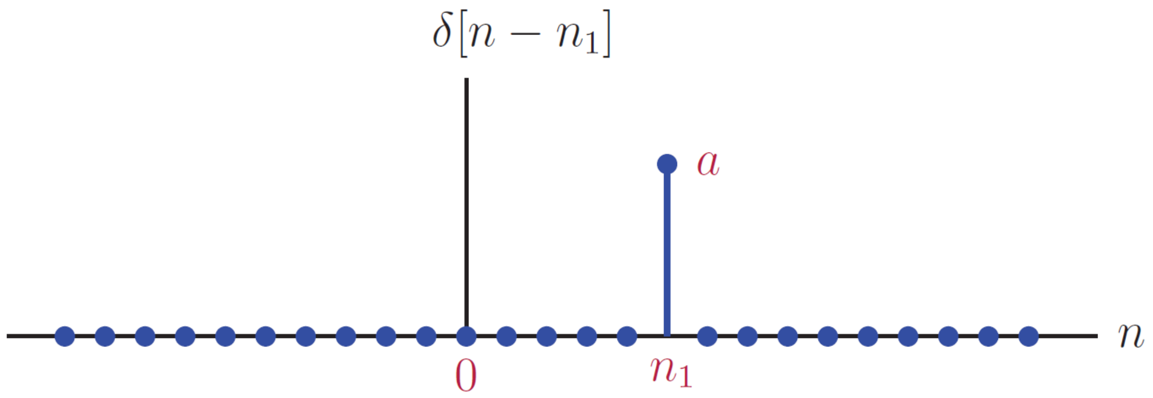
- The discrete-time unit-impulse function is defined by

$$\delta[n] = \begin{cases} 1\,, & n = 0 \\ 0\,, & n \neq 0 \end{cases}$$



- A unit-impulse function that is scaled by a and time shifted by n_1 samples is described by

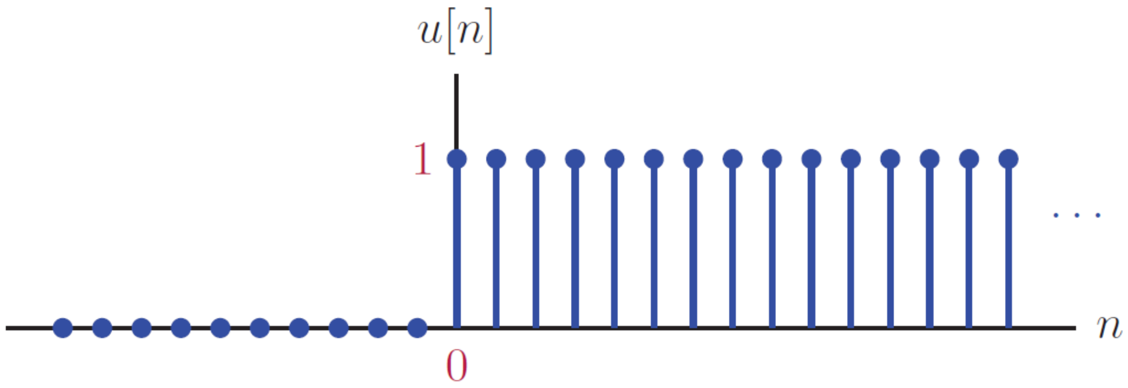
$$a\,\delta[n-n_1] = \begin{cases} a\,, & n = n_1 \\ 0\,, & n \neq n_1 \end{cases}$$



Unit-Step Function

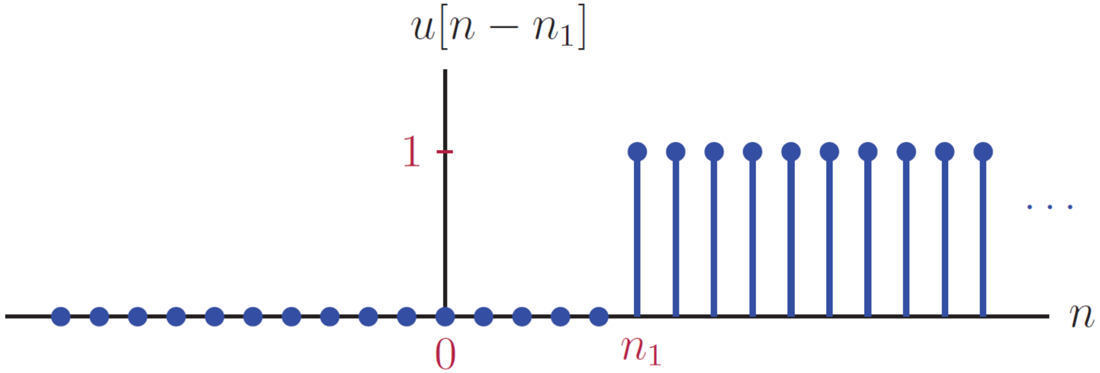
- The discrete-time version of the unit-ramp function is defined as

$$u[n] = \begin{cases} 1\,, & n \geq 0 \\ 0\,, & n < 0 \end{cases}$$



- A time shifted version of the discrete-time unit-step function can be written as

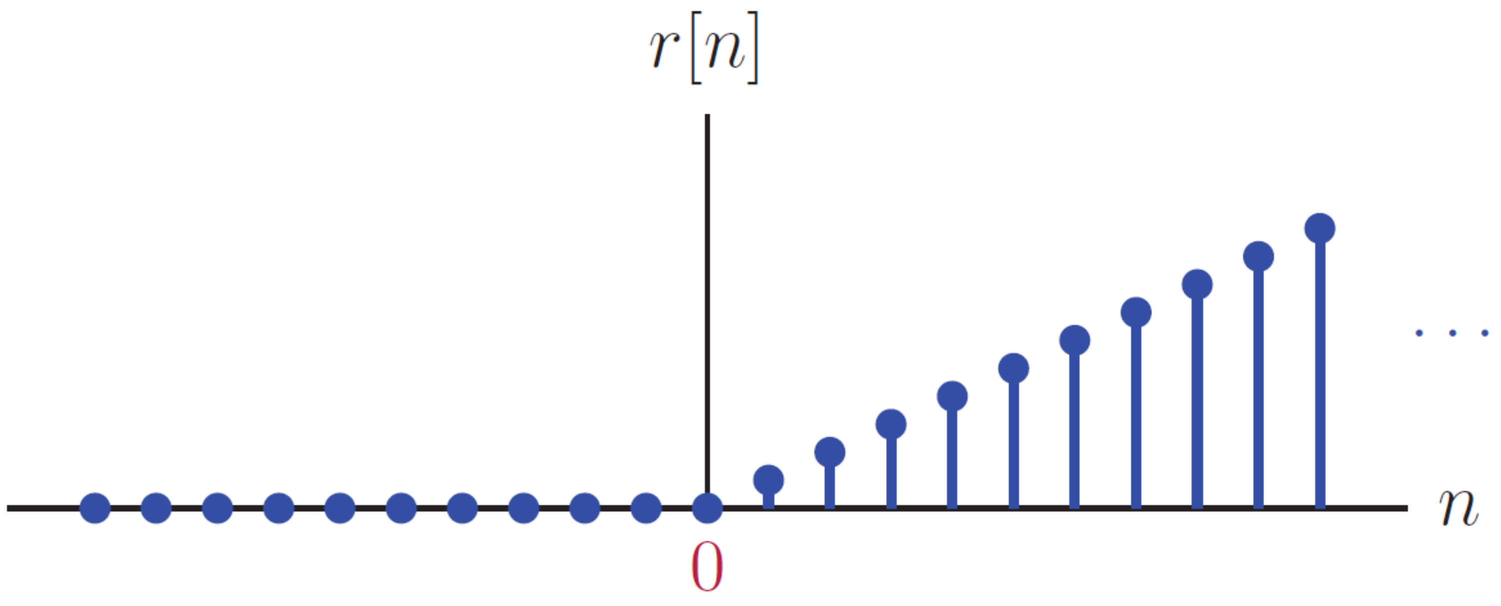
$$u[n-n_1] = \begin{cases} 1\,, & n \geq n_1 \\ 0\,, & n < n_1 \end{cases}$$



Unit-Ramp Function

- The discrete-time version of the unit-ramp function is defined as

$$r[n] = \begin{cases} n\ , & n \geq 0 \\ 0\ , & n < 0 \end{cases}$$



Discrete-Time Sinusoidal Signals

- A discrete-time sinusoidal signal is in the general form

$$x[n] = A\cos(\Omega_0 n + \theta)$$

- The parameter A is the amplitude.
- The parameter Ω_0 is the angular frequency in radians.

$$\Omega_0 = 2\pi F_0$$

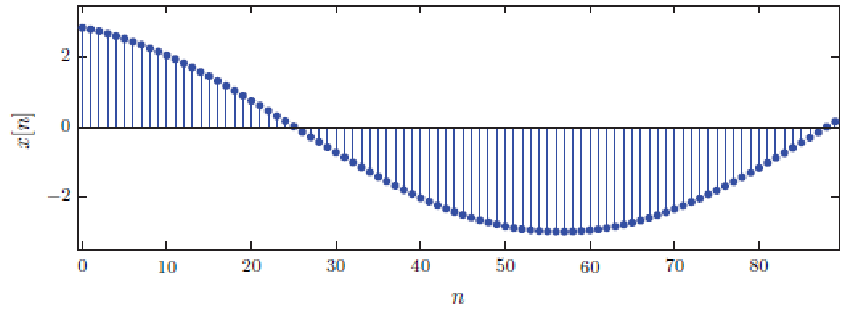
$$F_0 = \frac{\Omega_0}{2\pi}$$

- The parameter θ is the phase angle in radians.

Discrete-Time Sinusoidal Signals

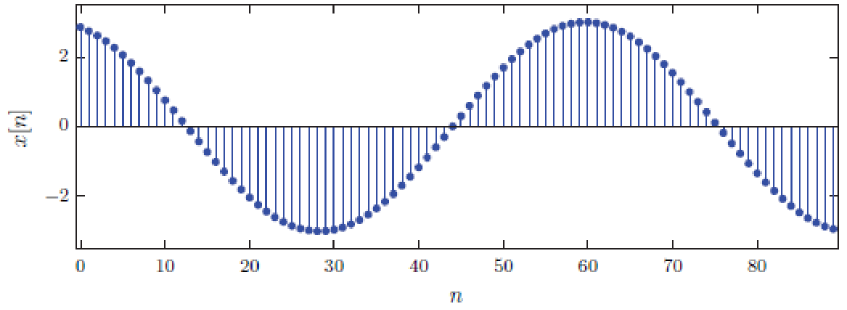
- Discrete-time sinusoidal signal $x[n] = 3\cos(\Omega_0 n + \pi/10)$ for

(a) $\Omega_0 = 0.05$ rad



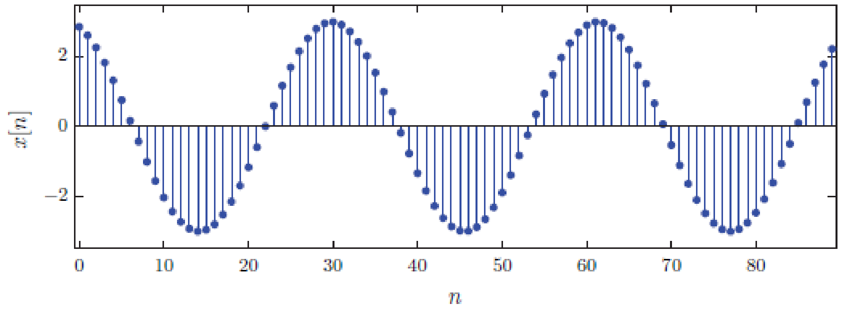
(a)

(b) $\Omega_0 = 0.1$ rad



(b)

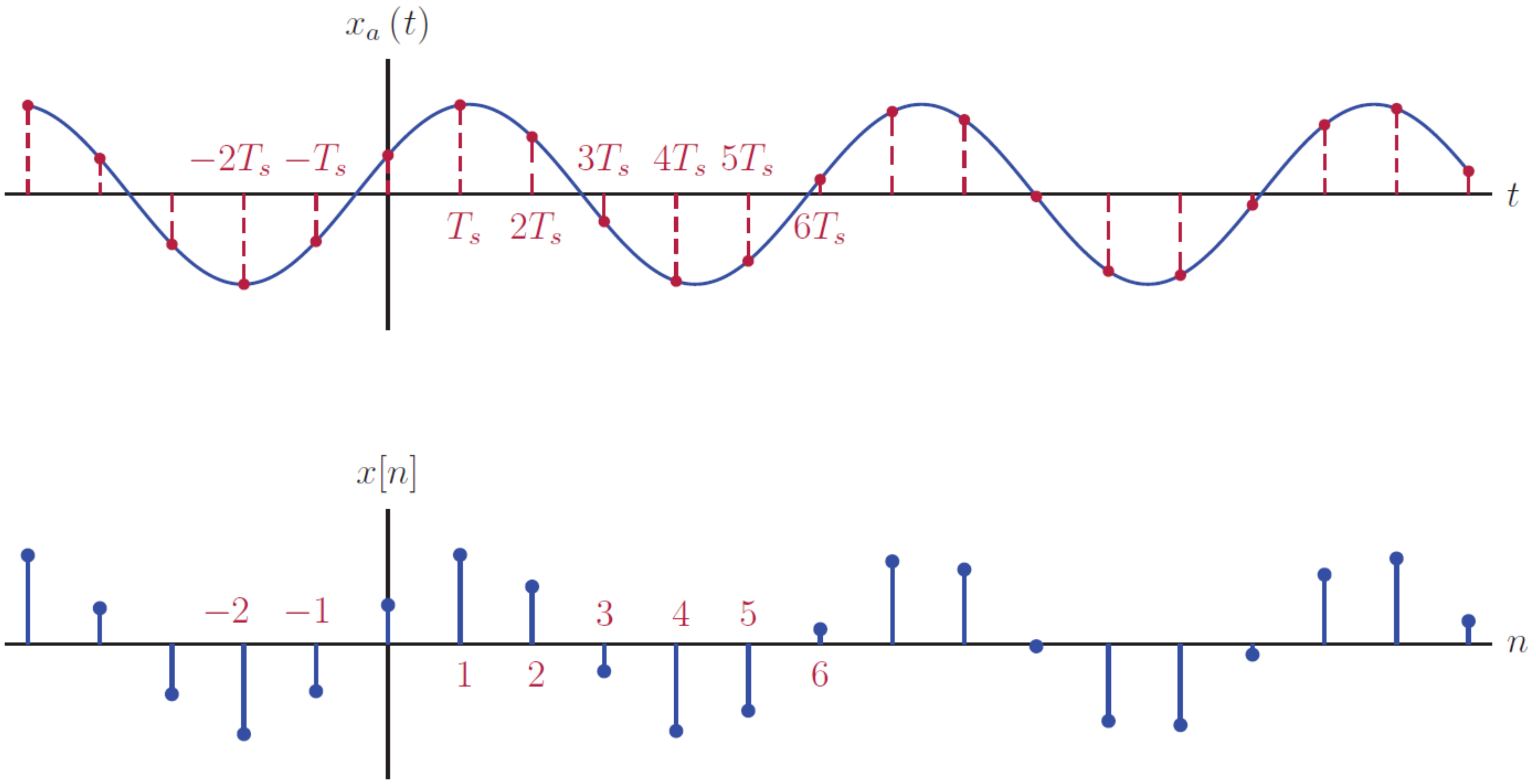
(c) $\Omega_0 = 0.2$ rad



(c)

Discrete-Time Sinusoidal Signals

- Obtaining a discrete-time sinusoidal signal from a continuous-time sinusoidal signal.

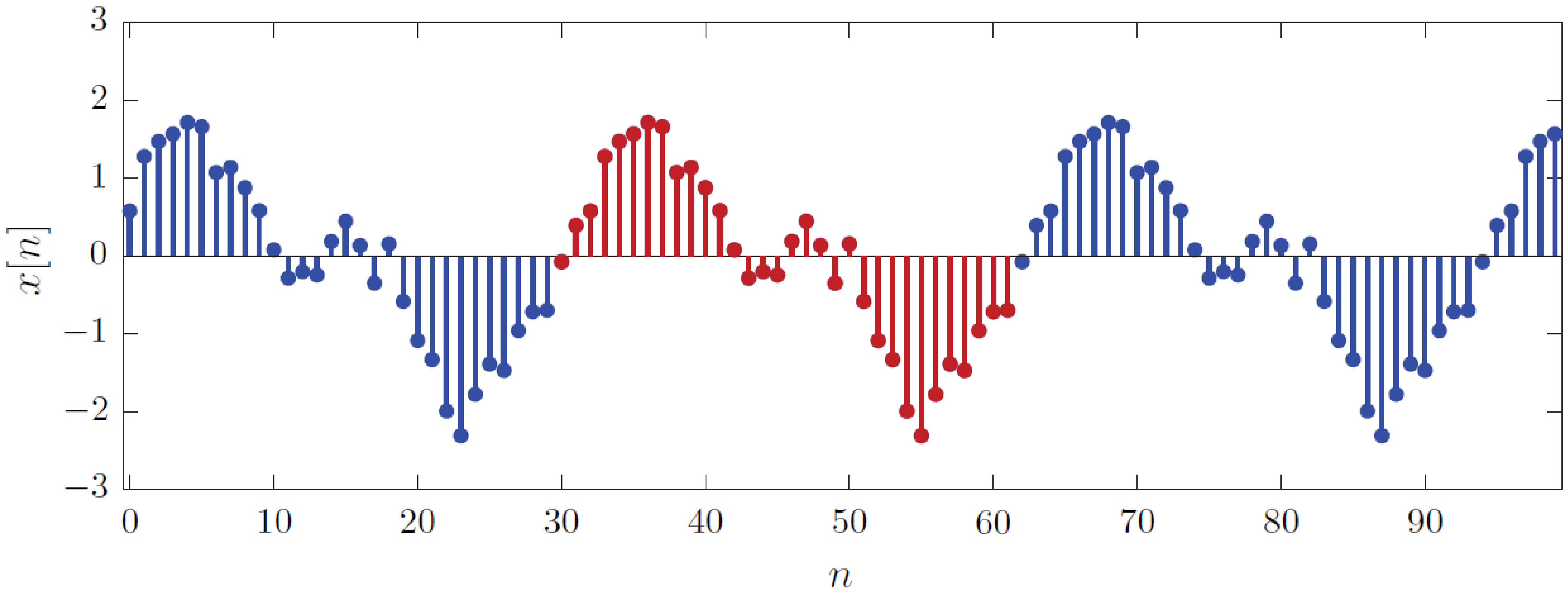


Periodic vs. Non-Periodic Signals

- A discrete-time signal is said to be periodic if it satisfies

$$x[n] = x[n + N]$$

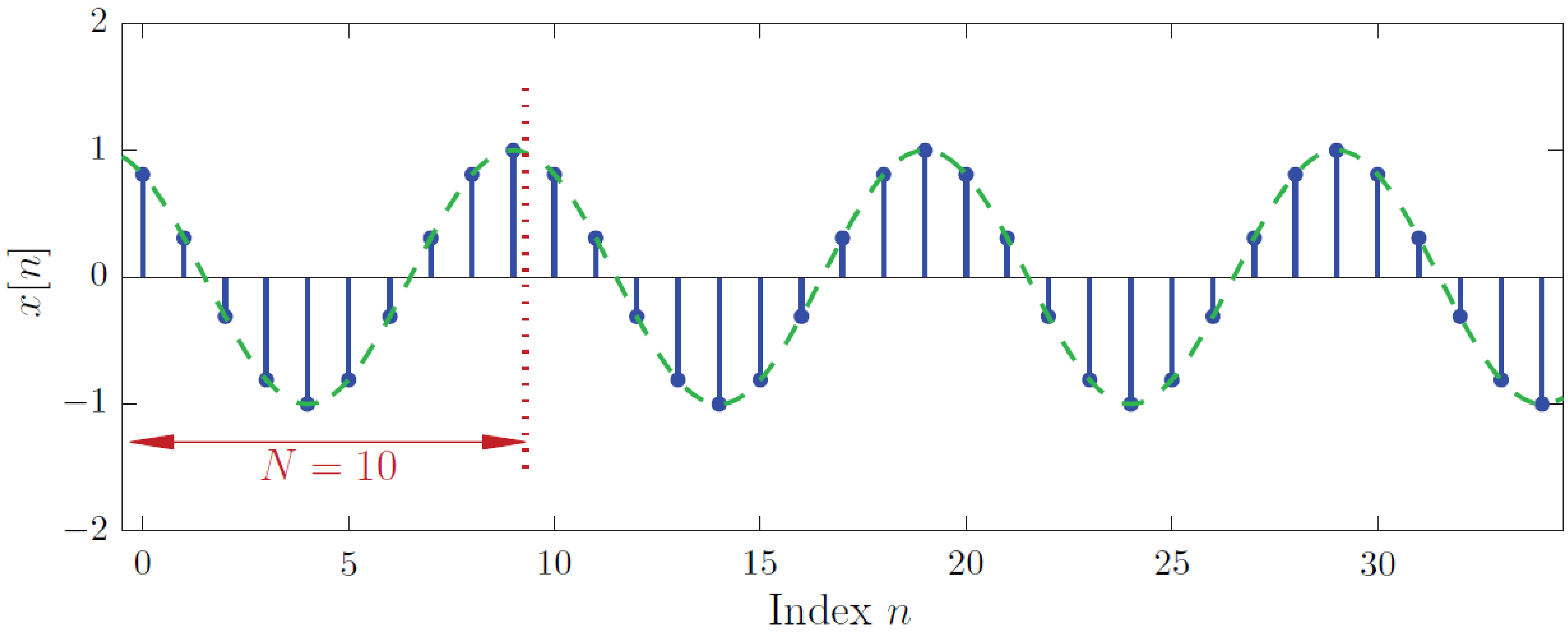
for all values of the integer index n and for a specific value of $N \neq 0$.



Periodic vs. Non-Periodic Signals

- The parameter N is referred to as the period of the signal.

$$F_0 = \frac{1}{N}$$



Periodic vs. Non-Periodic Signals

- A discrete-time signal that is periodic with a period of N samples is also periodic with periods of $2N, 3N, \ldots, kN$ for any positive integer k.
- For the sinusoidal signal $x[n]$ to be periodic, it needs to satisfy

$$2\pi F_0 N = 2\pi k$$

and consequently

$$N = \frac{k}{F_0}$$

- Since we are dealing with a discrete-time signal, there is the added requirement that the period N obtained must be an integer value.

Example 1.16

Check the periodicity of the following discrete-time signals:

a. $x[n] = \cos(0.2n)$

b. $x[n] = \cos(0.2\pi n + \pi/5)$

c. $x[n] = \cos(0.3\pi n - \pi/10)$

Example 1.16 (a) – Solution

a. $x[n] = \cos(0.2n)$

a. The angular frequency of this signal is $\Omega_0 = 0.2$ radians which corresponds to a normalized frequency of

$$F_0 = \frac{\Omega_0}{2\pi} = \frac{0.2}{2\pi} = \frac{0.1}{\pi}$$

This results in a period

$$N = \frac{k}{F_0} = 10\pi k$$

Since no value of k would produce an integer value for N, the signal is not periodic.

Example 1.16 (b) – Solution

b. $x[n] = \cos\left(0.2\pi n + \pi/5\right)$

b. In this case the angular frequency is $\Omega_0 = 0.2\pi$ radians, and the normalized frequency is $F_0 = 0.1$. The period is

$$N = \frac{k}{F_0} = \frac{k}{0.1} = 10k$$

For $k = 1$ we have $N = 10$ samples as the fundamental period. The signal $x[n]$ is shown in Fig. 1.83.

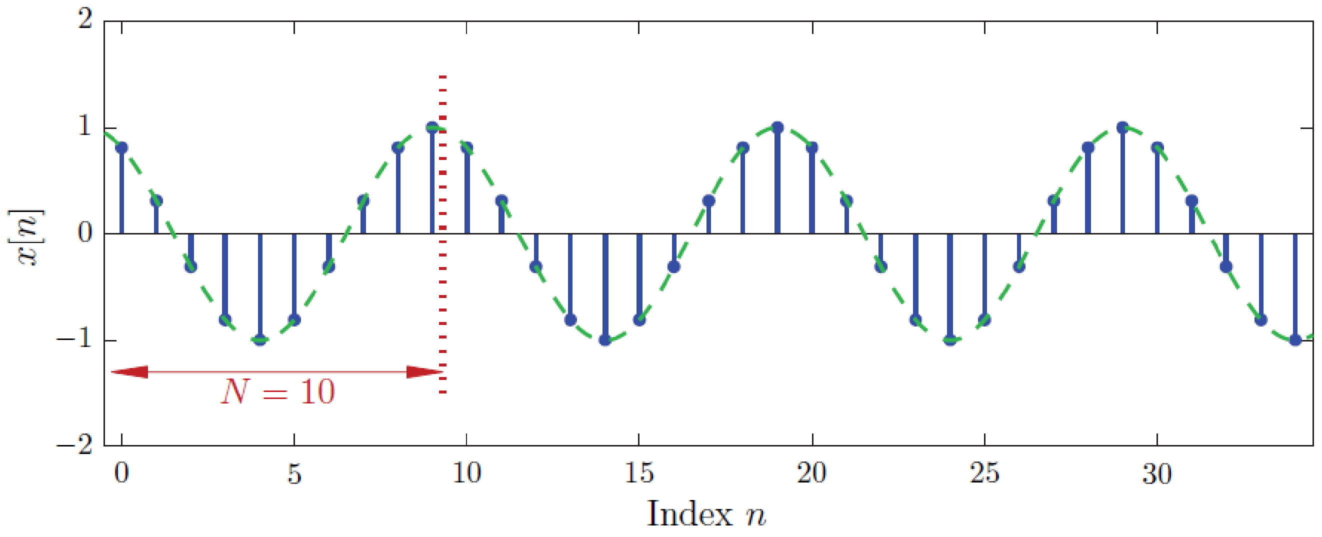


Figure 1.83 – The signal $x[n]$ for part (b) of Example 1.16.

Example 1.16 (c) – Solution

c. $\quad x[n] = \cos\left(0.3\pi n - \pi/10\right)$

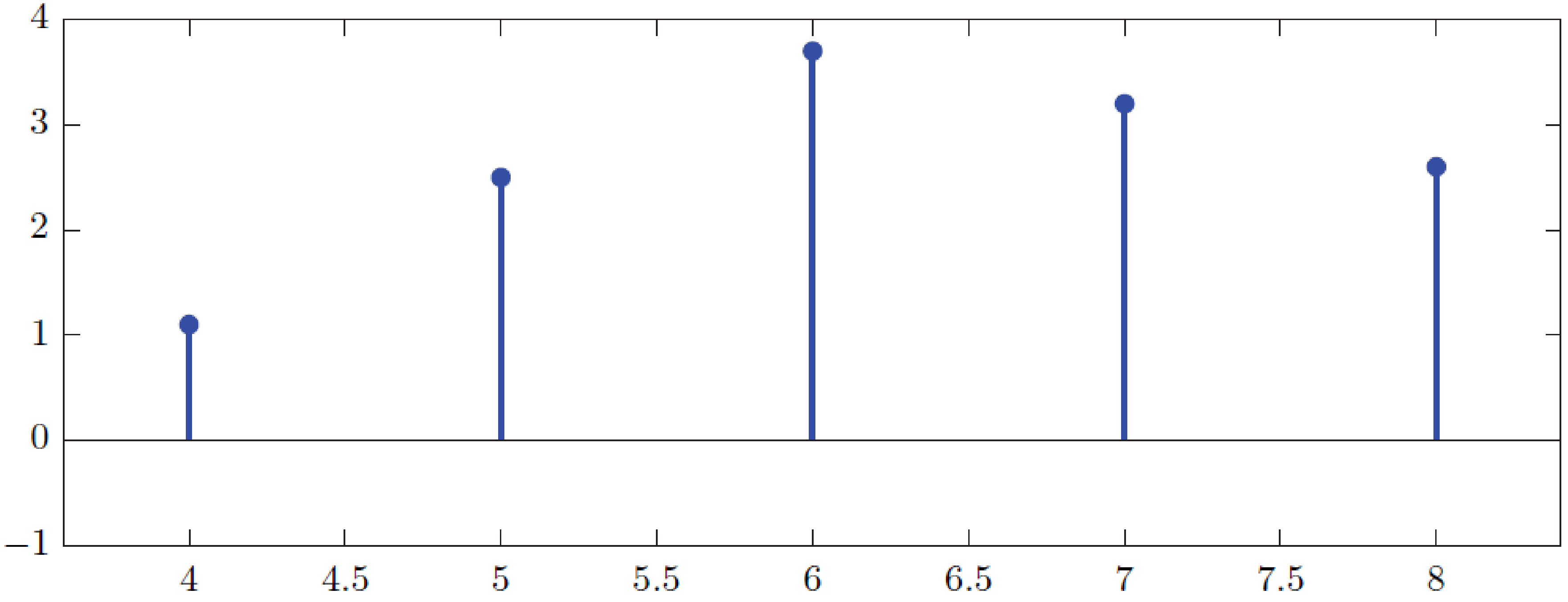
c. For this signal the angular frequency is $\Omega_0 = 0.3\pi$ radians, and the corresponding normalized frequency is $F_0 = 0.15$. The period is

$$N = \frac{k}{F_0} = \frac{k}{0.15}$$

The smallest positive integer k that would result in an integer value for the period N is $k = 3$. Therefore, the fundamental period is $N = 3/0.15 = 20$ samples. The signal $x[n]$ is shown in Fig. 1.84.

Computing and Graphing Discrete-Time Signals

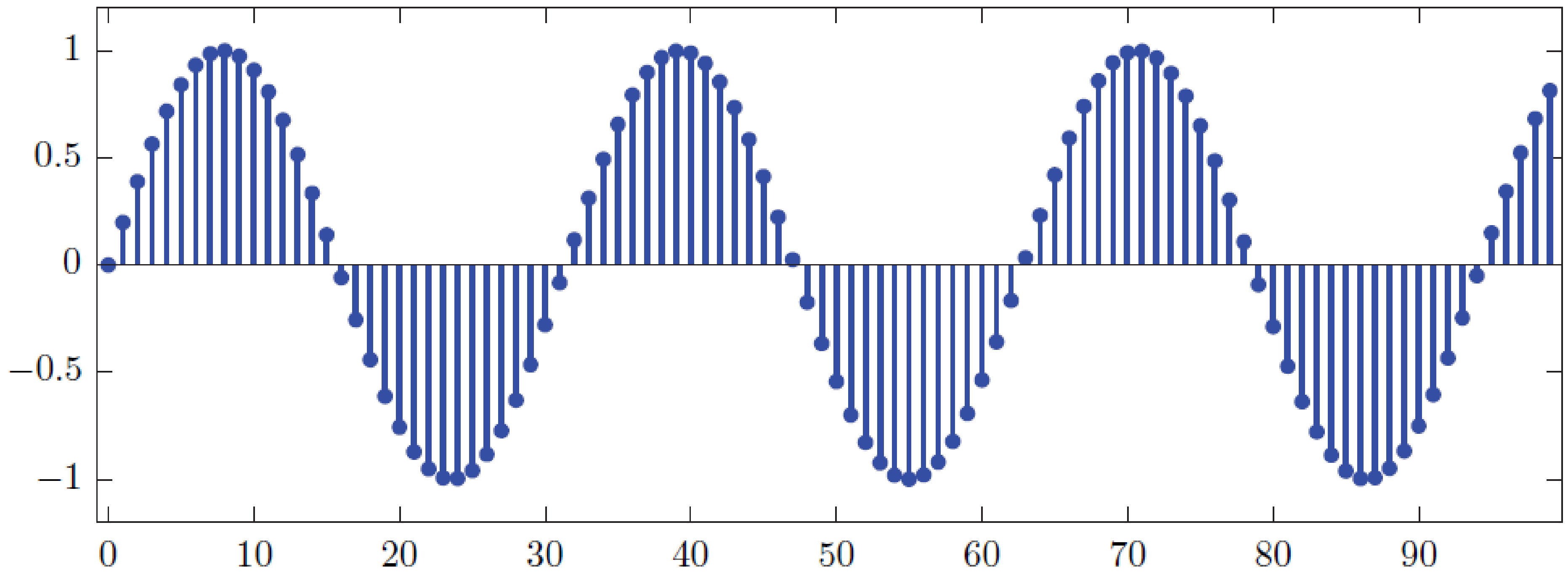
```
n = 4:8;
x = [1.1, 2.5, 3.7, 3.2, 2.6];
stem(n, x);
```



Computing and Graphing Discrete-Time Signals

```
n = 0:99;
x2 = sin (0.2* n);
stem(n, x);
```

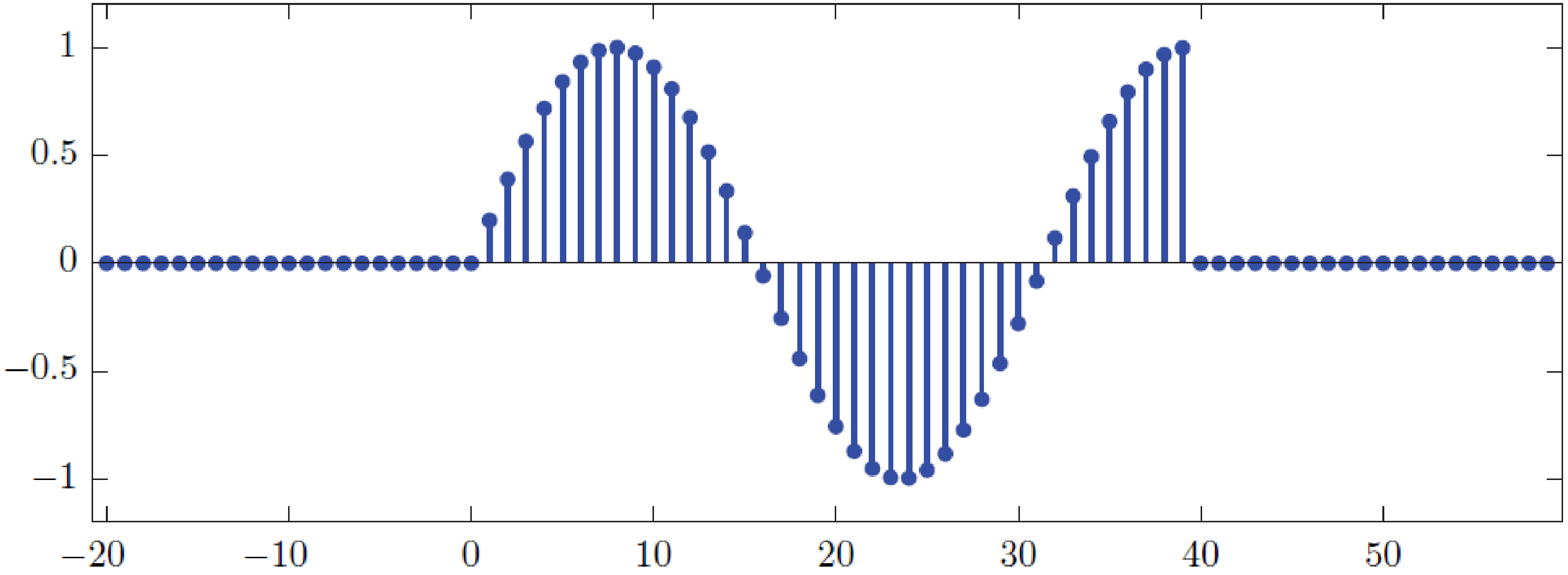
$$x[n] = \sin(0.2n)$$



Computing and Graphing Discrete-Time Signals

```
n = -20:59;
x = sin (0.2* n).*((n >=0)&( n <=39));
stem(n, x);
```

$$x_3[n] = \begin{cases} \sin(0.2n) \ , & n = 0, \ldots, 39 \\ 0 \ , & \text{otherwise} \end{cases}$$



Problem 1.47

1.47. Consider the discrete-time signal $x[n]$ used in Problem 1.33 and graphed in Fig. P.1.33.

a. Express this signal through an anonymous MATLAB function that utilizes the function `ss_ramp(..)`, and graph the result for index range $n = -10, \ldots, 10$.

b. Express each of the signals in parts (a) through (h) of Problem 1.33 in MATLAB, and graph the results. Use functions `ss_step(..)` and `ss_ramp(..)` as needed.

a. $g[n] = x[n-3]$

b. $g[n] = x[2n-3]$

c. $g[n] = x[-n]$

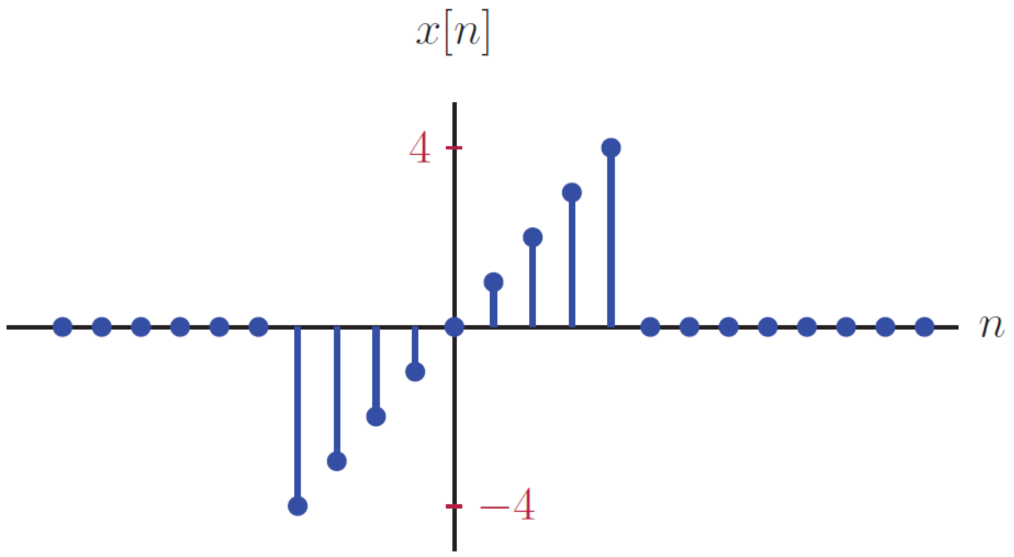
d. $g[n] = x[2-n]$

e. $g[n] = \begin{cases} x[n/2] \,, & \text{if } n/2 \text{ is integer} \\ 0 \,, & \text{otherwise} \end{cases}$

f. $g[n] = x[n]\,\delta[n]$

g. $g[n] = x[n]\,\delta[n-3]$

h. $g[n] = x[n]\,\{u[n+2] - u[n-2]\}$



Problem 1.47 – Solution

SigSys_MATLAB_v1_03b\SigSys\MATLAB_Code\Chapter01

ss_step.m

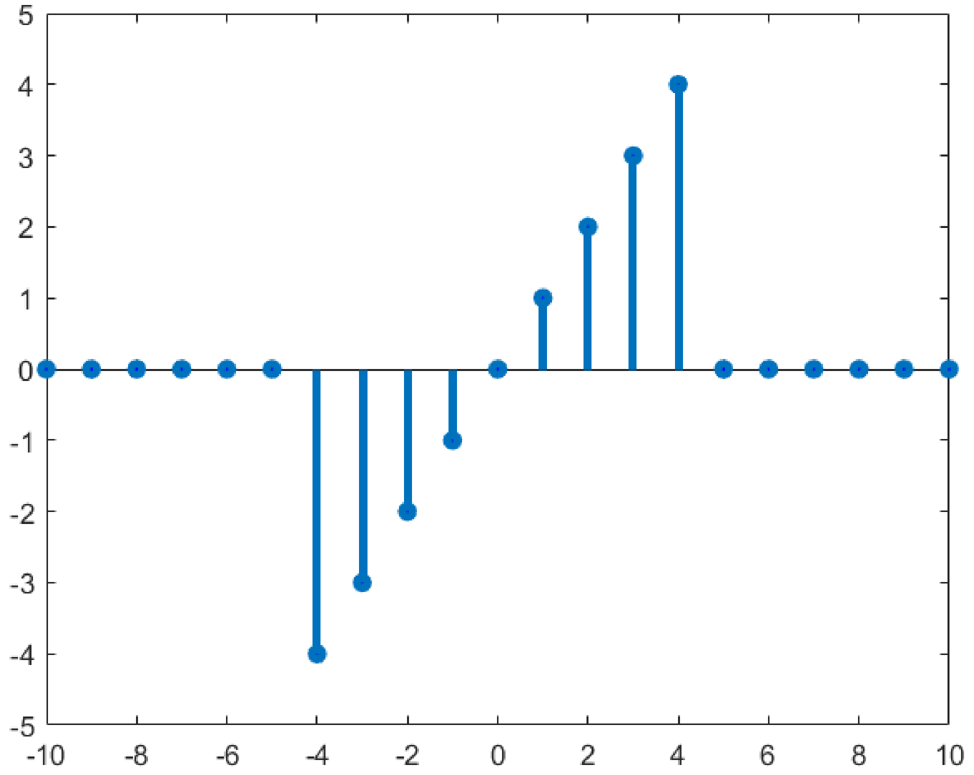
```
function x = ss_step(t)
  x = 1*(t>=0);
```

ss_ramp.m

```
function x = ss_ramp(t)
  x = t.*(t>=0);
```

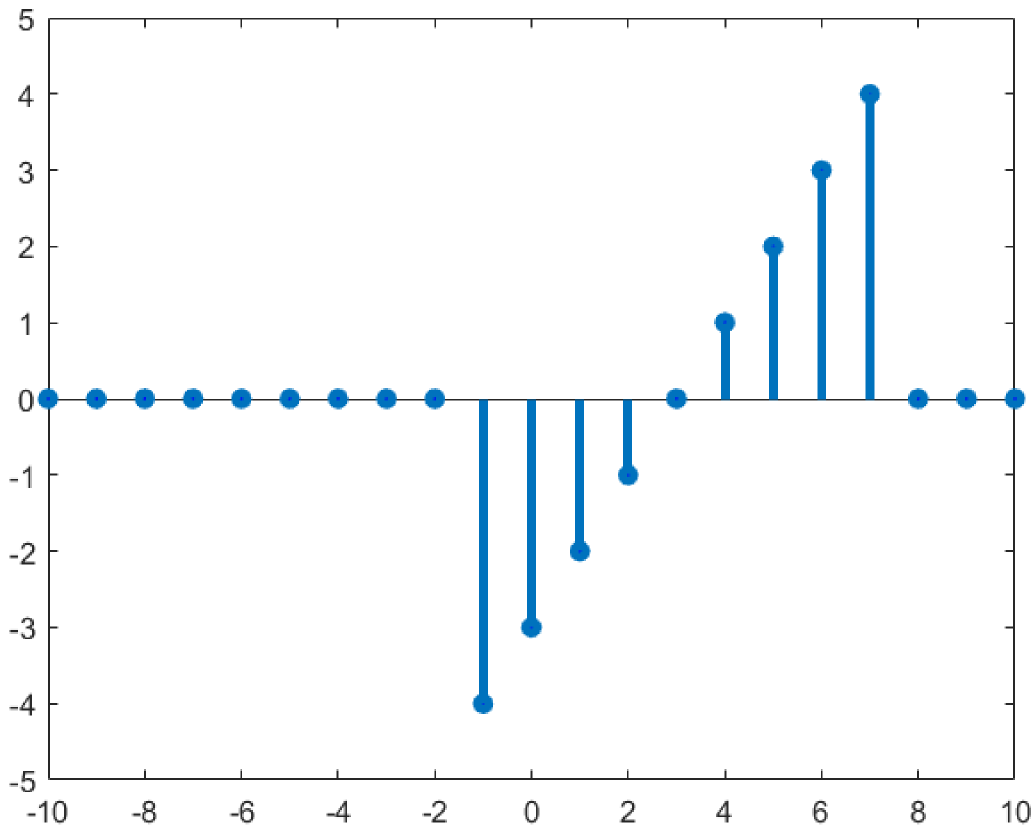
Problem 1.47 (a) – Solution

```
n = -10:10;
x = @(n) n .* ((n>=-4) & (n<=4));
stem(n, x(n));
```



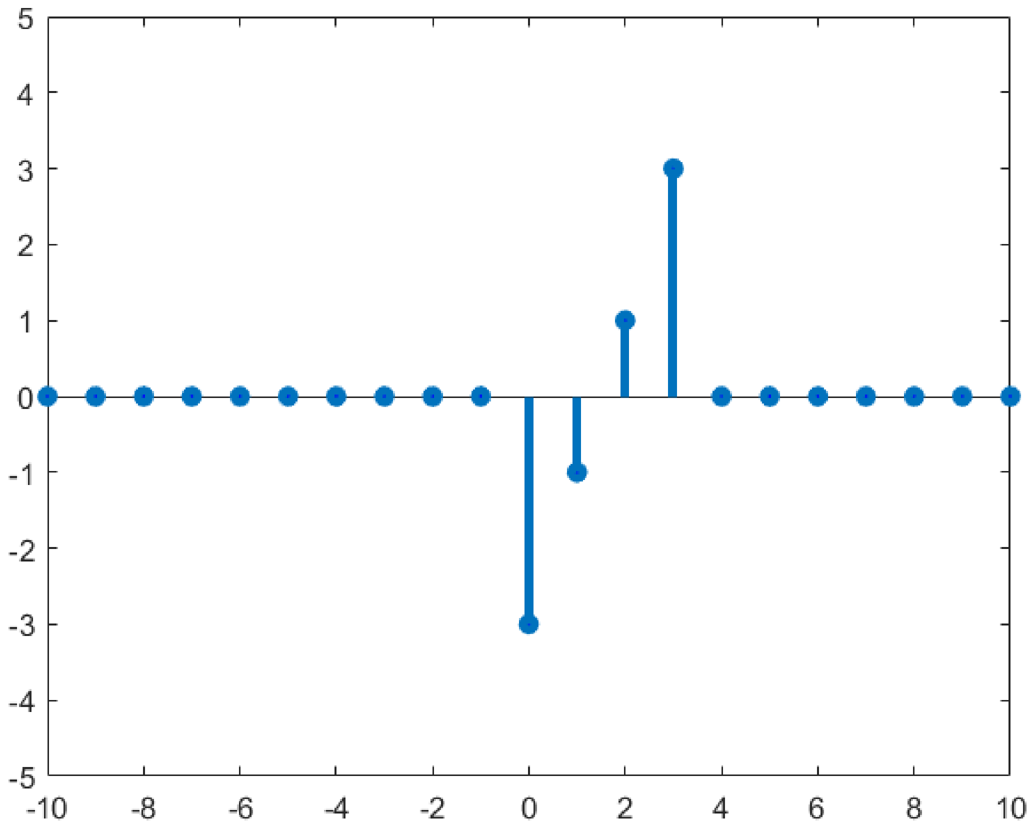
Problem 1.47 (b) – Solution

```
g1 = x(n - 3);
stem(n, g1);
```



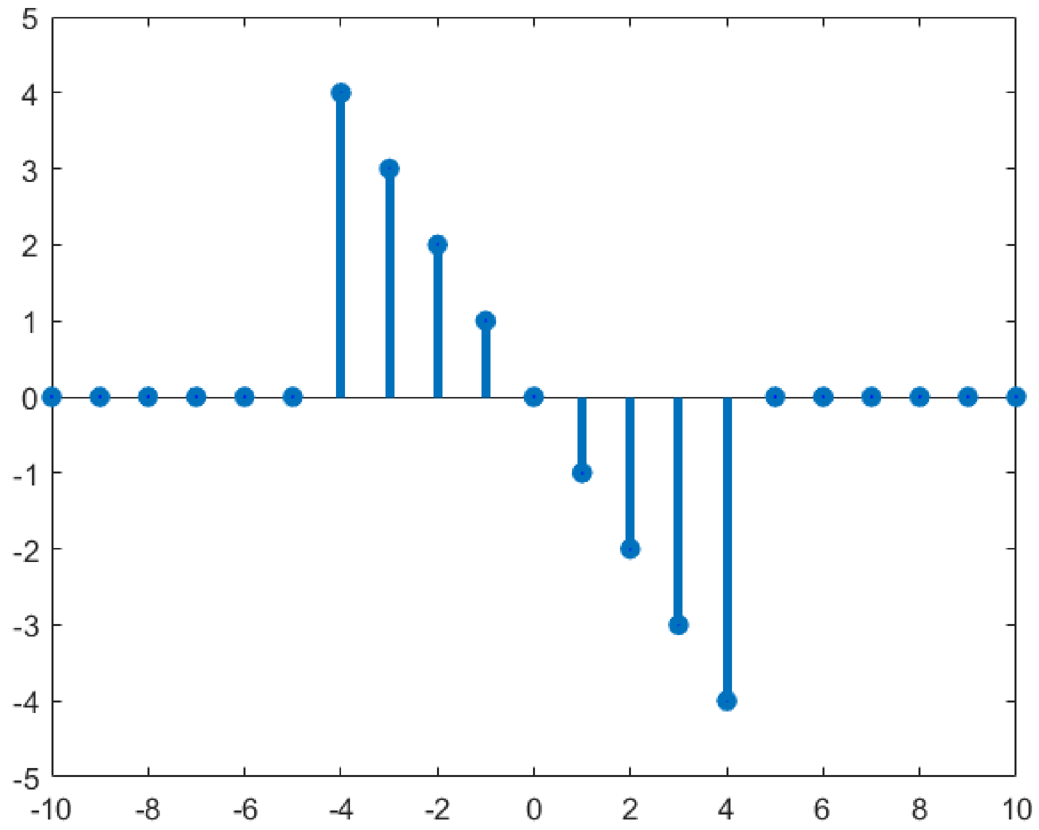
Problem 1.47 (b) – Solution

```
g2 = x(2*n - 3);
stem(n, g2);
```



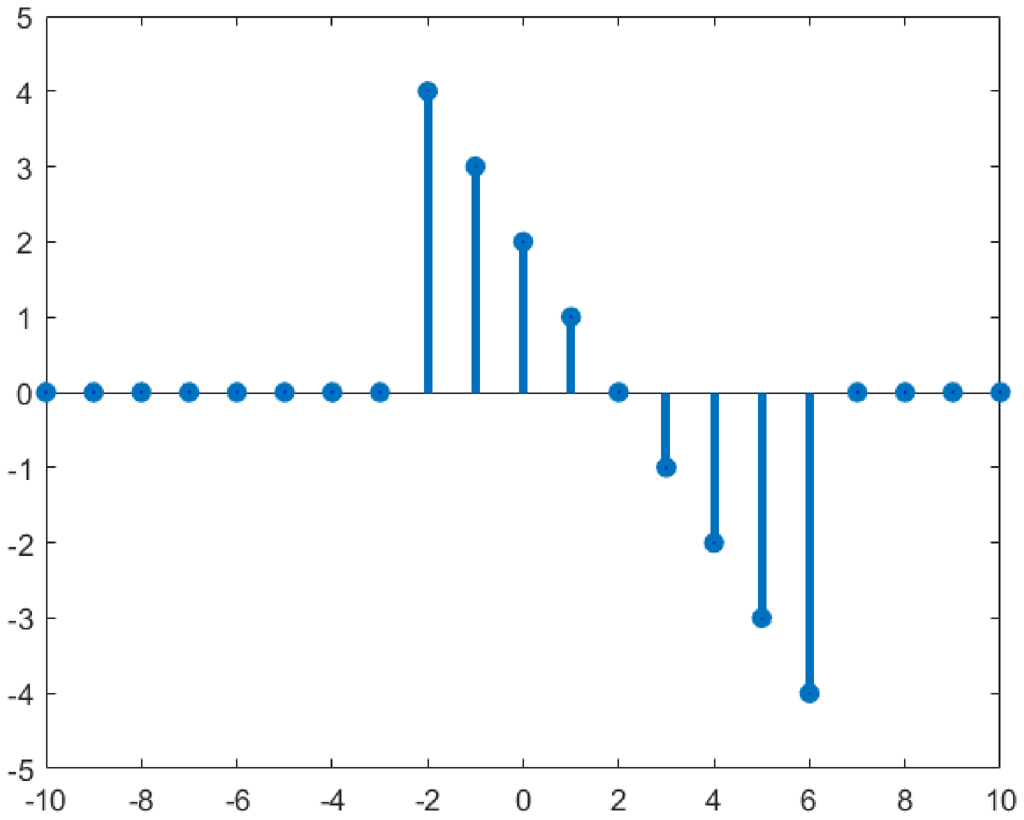
Problem 1.47 (b) – Solution

```
g3 = x(-n);
stem(n, g3);
```



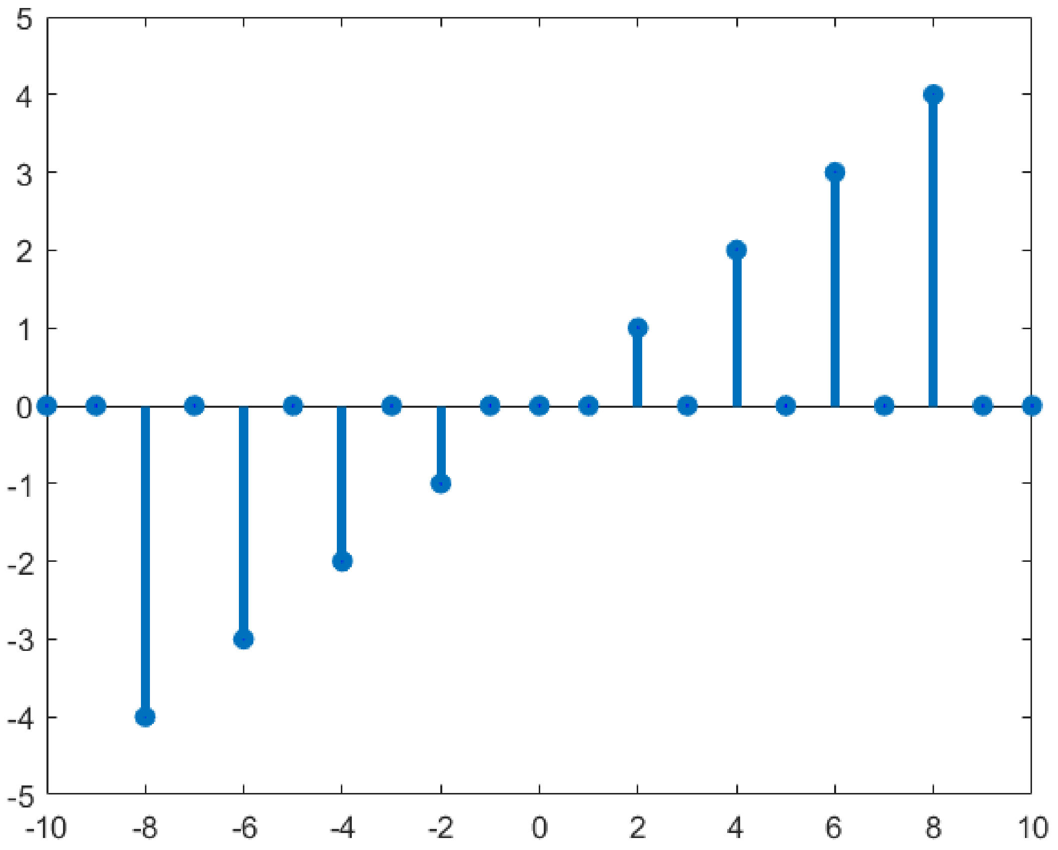
Problem 1.47 (b) – Solution

```
g4 = x(2 - n);
stem(n, g4);
```



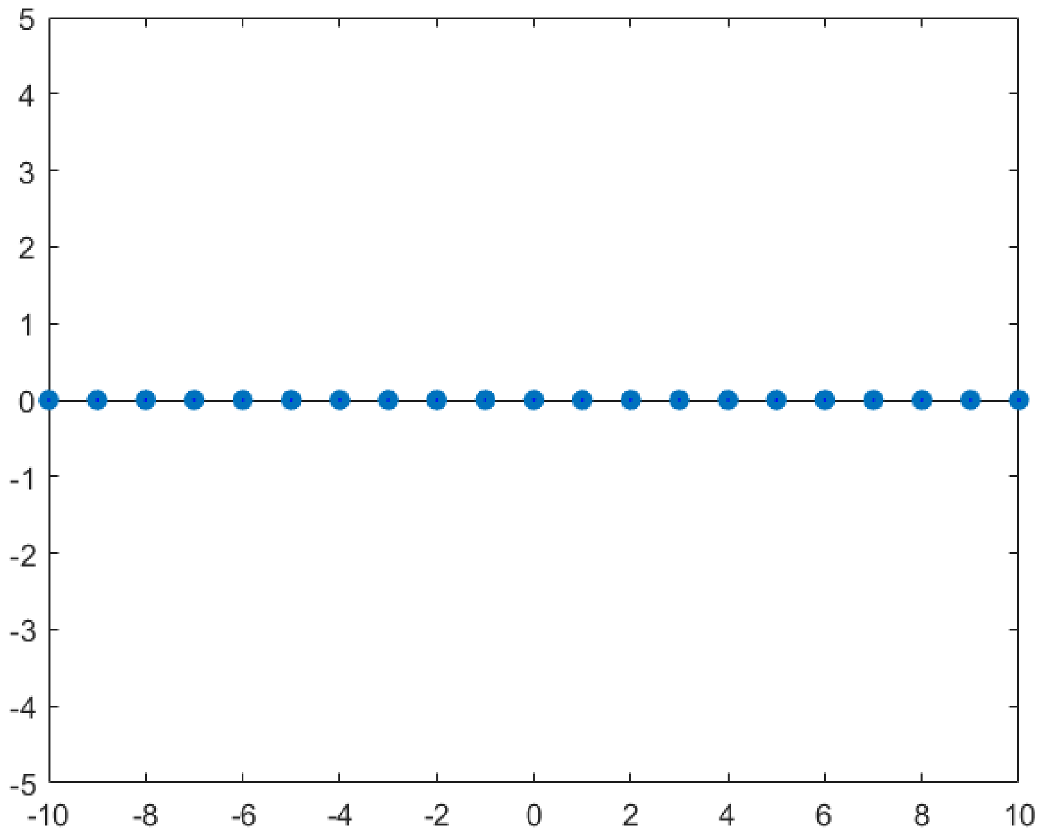
Problem 1.47 (b) – Solution

```
g5 = x(n/2) .* (mod(n, 2)==0);
stem(n, g5);
```



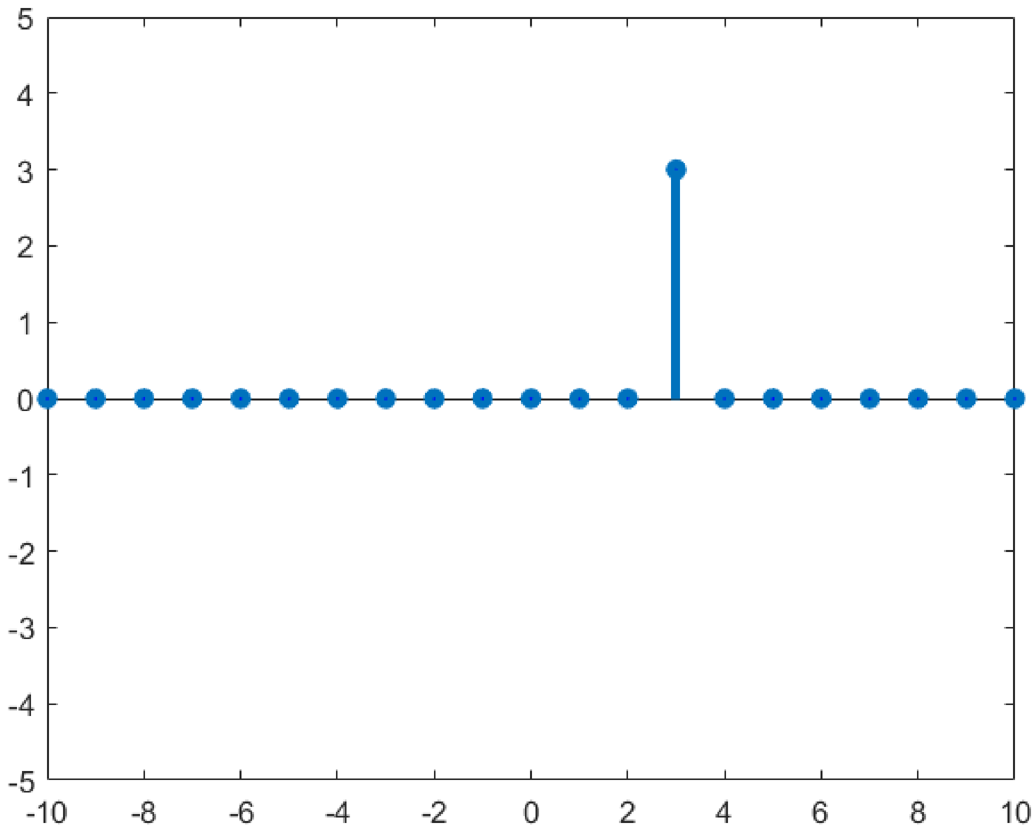
Problem 1.47 (b) – Solution

```
g6 = x(n) .* (n==0);
stem(n, g6);
```



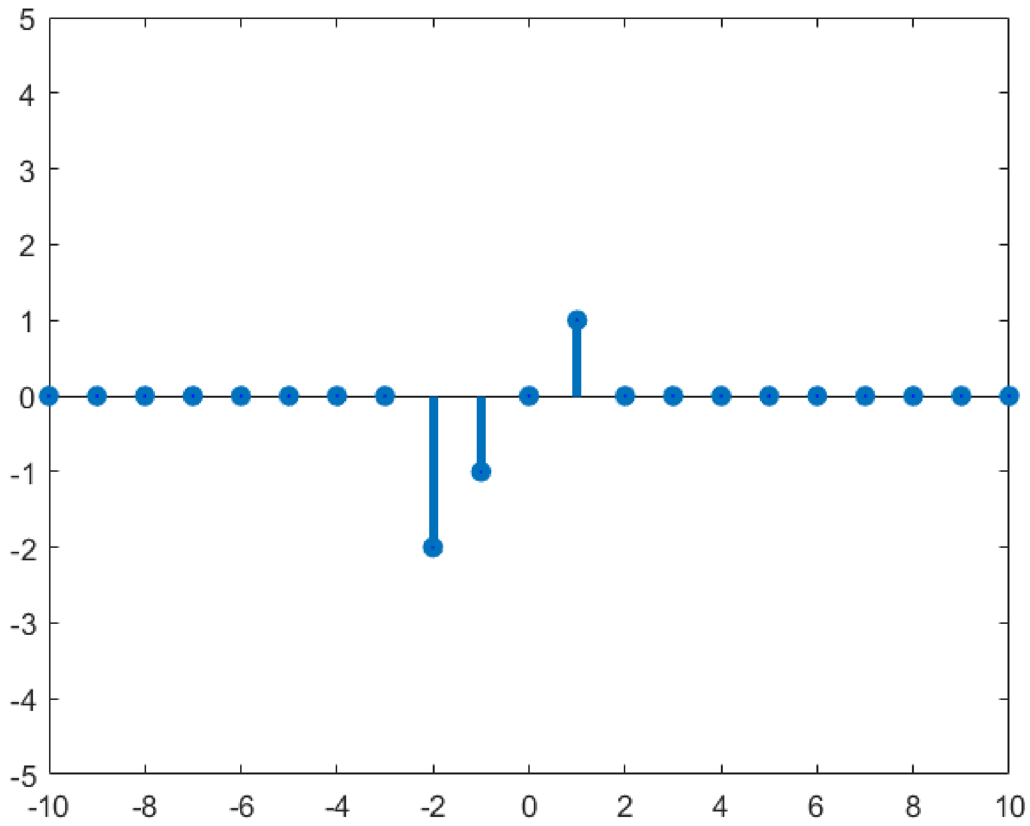
Problem 1.47 (b) – Solution

```
g7 = x(n) .* (n==3);
stem(n, g7);
```



Problem 1.47 (b) – Solution

```
g8 = x(n) .* (ss_step(n+2) - ss_step(n-2));
stem(n, g8);
```



Chapter 1: Interactive Demos

```
>> sop_demo1
>> sop_demo2
>> wav_demo1
>> stp_demo1
>> sin_demo1
>> tavg_demo
```

Appendix: Definite Integrals

1. *Order of Integration:* $$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$ A definition

2. *Zero Width Interval:* $$\int_a^a f(x)\,dx = 0$$ A definition when $f(a)$ exists

3. *Constant Multiple:* $$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$$ Any constant k

4. *Sum and Difference:* $$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

5. *Additivity:* $$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$